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LOW ALTITUDE DEFENSE:
AN ANALYSIS OF ITS EFFECT
ON MX SURVIVABILITY.

THESIS,

James T./Moore
AFIT/GST/OS/81M-10   Capt   USAF

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This thesis investigates MX survivability when a terminal ballistic missile defense system known as Low Altitude Defense (LOAD) is deployed. LOAD will defend the MX missile with three high-speed, nuclear-armed interceptor missiles. This research determines a best strategy for the use of the interceptors. The deployment of LOAD with its best strategy is compared to increases in MX shelter hardness to determine which is the more effective method of improving MX survivability.

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LOW ALTITUDE DEFENSE:

AN ANALYSIS OF ITS EFFECT

ON MX SURVIVABILITY

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Capt   USAF

Graduate Strategic and Tactical Sciences

March 1981

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Preface

John Crandley, classmate and close friend, deserves
special thanks for his contributions to this thesis. He
suggested that we do a project in the area of BMD for
Military Systems Simulations and that project served as
the cornerstone of this effort. The encouragement and
advice of Dave Lee and Joe Alt were a significant aid to
me in the completion of the thesis. Dan Fox and Tom Clark,
my advisor and reader, were supportive and patient with me;
to them, I say thank you.

(This thesis was typed by Sharon A. Gabriel)

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This thesis investigates MX survivability when a terminal ballistic missile defense system known as Low Altitude Defense (LOAD) is deployed. LOAD will defend the MX missile with three high-speed, nuclear-armed interceptor missiles. This research determines a best strategy for the use of the interceptors. The deployment of LOAD with its best strategy is compared to increases in MX shelter hardness to determine which is the more effective method of improving MX survivability.

LOW ALTITUDE DEFENSE:

AN ANALYSIS OF ITS EFFECT

ON MX SURVIVABILITY

I. Introduction

In the 1980s, the United States will develop and
deploy a new intercontinental ballistic missile (ICBM).
This missile, designated the MX, is the first American
land-based ICBM to use a deceptive basing scheme. The
rationale for such a basing scheme is obvious: the missile
is not easily destroyed if its exact location is not known.

Plans are currently being made to build and deploy
200 MX missiles in the desert valleys of Western Utah and
Eastern Nevada. Within each valley, there will be a
number of straight line tracks. Each track will connect
23 shelters for the one MX missile deployed in it (see
Figure 1). The key feature of this system will be "preser-
vation of location uncertainty" (PLU). PLU will encompass
many carefully designed procedures which will prevent the
enemy from determining which of the 23 shelters contains
the MX. However, the enemy will be able to determine the
exact location of each shelter (Refs 15, 26).

In the United States, the Army is charged with the
defense of ICBMs and has examined the problem of assuring
an acceptable level of MX survivability. If PLU is

1
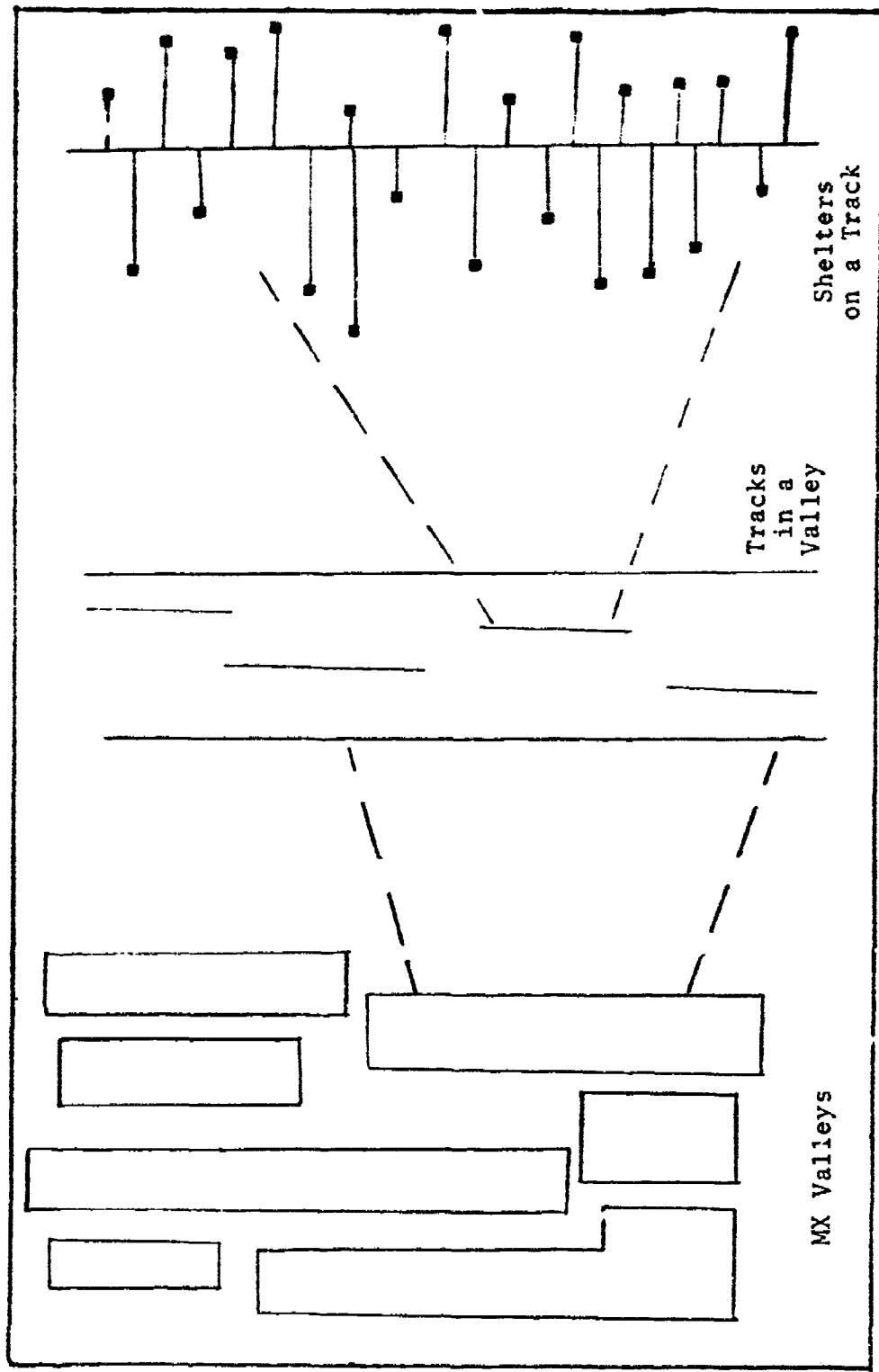
Figure 1.   Physical Set-Up of MX System

successful and the enemy does not know which of the 23
shelters of an MX complex contains the MX, then the probab-
ility of a single enemy reentry vehicle (RV) destroying
the MX cannot exceed one in 23. However, as the number of
enemy RVs targeted on an MX complex increases, the probab-
ility of MX survival decreases. As Soviet RV technology
improves, the effectiveness of individual RVs increases and,
with sufficient yield and accuracy, the probability of one
enemy RV being able to destroy one MX shelter will approach
one. 4600 attacking RVs would then be able to destroy
4600 MX shelters and the 200 MX missiles they harbor.
Because of this threat, the Army is exploring ballistic
missile defenses (BMDs).

The Army has studied several types of BMDs which could
improve the survivability of MX. One area of emphasis in
BMD has been the development of a terminal defense system
for the MX. Terminal defense occurs during the reentry
phase of an attacking RV's trajectory. Emphasis has been
placed on the terminal regime because the atmosphere helps
the defense. The atmosphere filters out non-threatening
objects, provides wake observables to aid in discrimination,
and slows down RVs. The terminal regime is characterized
by a severely compressed timeline. There are only 15 seconds
for the terminal defense to acquire, track, and intercept
the attacking RV. This environment places strict require-
ments on the radar, computer, and interceptor of the terminal
defense system (Ref 20).

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Terminal defense is ideally suited to the defense of MX because of the leverage gained. Leverage is defined as the ratio of the number of RVs in the threat to the number of interceptors required to satisfy defense objectives (Ref 20:39). The deployment of one interceptor with each MX could double the number of RVs required to destroy MX (Ref 20:46).

One terminal defensive system the Army is evaluating is Low Altitude Defense (LOAD). LOAD would involve the placement of several nuclear-armed, very-high-speed interceptor missiles on a defensive unit (DU) which would be placed in one of the 23 MX shelters (Figure 2). Current plans call for the placement of the DU in a shelter close to the MX shelter, and in the event of an enemy attack, the DU's radar system would be used to determine which RVs were aimed at the MX shelter and/or the DU shelter. Interceptors would be launched at these RVs, but RVs aimed at empty shelters would not be intercepted (Ref 8).

Since each DU is expected to have three interceptors (Ref 25), a defensive strategy must be chosen which will provide the highest expected probability of MX survival. Three strategies are available. One strategy would require the use of all three interceptors for MX defense. Only RVs aimed at the MX shelter would be intercepted. Use of the first interceptor to defend either the MX or DU shelter is another possible strategy. The two remaining interceptors

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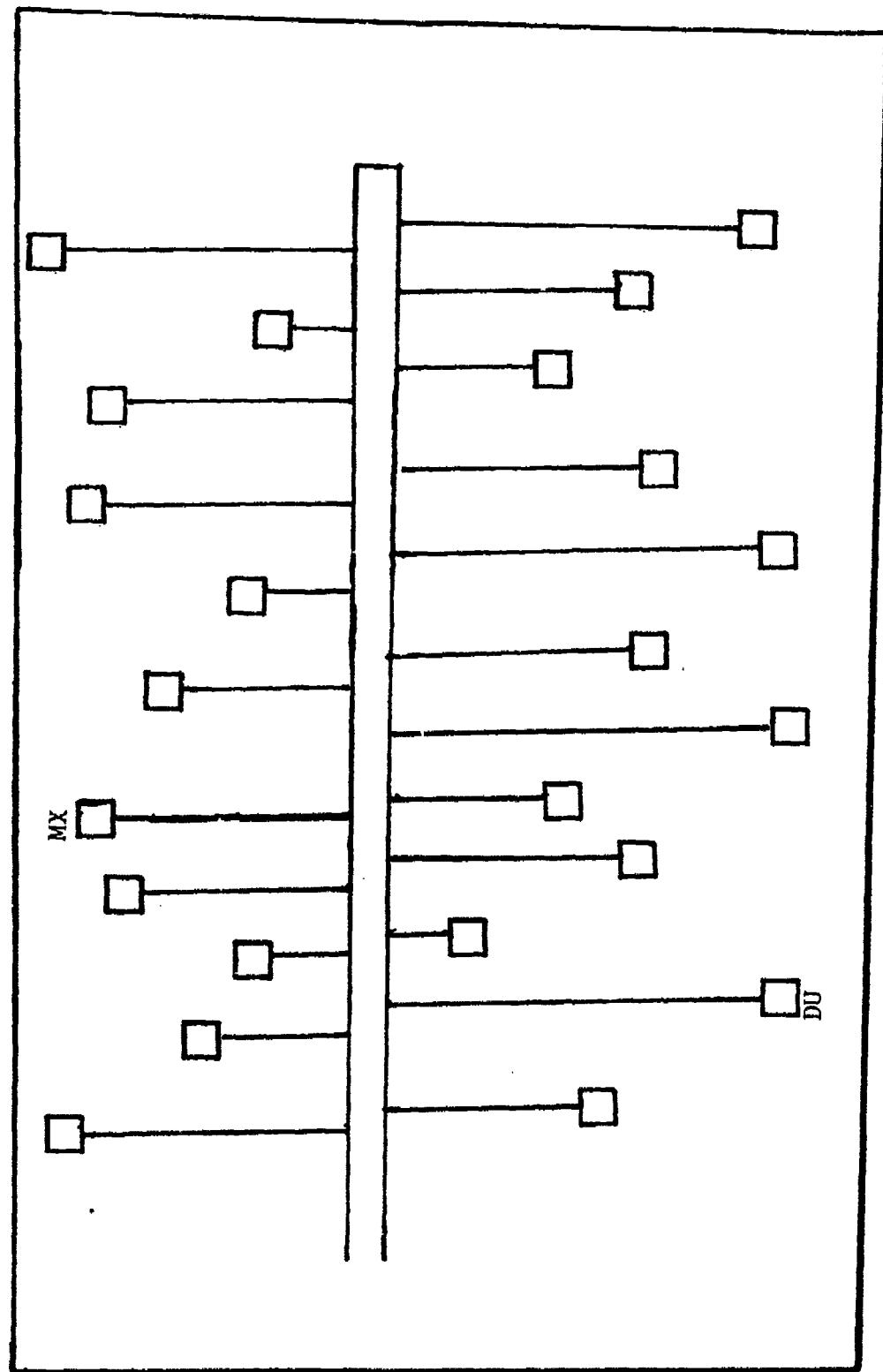
Figure 2. Random Assignment of DU and MX Launchers

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would be used for the defense of the MX shelter. A third strategy would use the first two interceptors for defense of the MX or DU shelter and use the last interceptor for MX defense.

Variations in the enemy offensive strategy would involve the attack of the MX missiles with increasing numbers of RVs. Three methods can be used to accomplish this strategy. One method would be the launch of more missiles with RVs targeted on the MX shelters. Reloading silos and launching additional missiles from them is another method. The third method is fractionization. Fractionization is the placement of additional RVs on a single missile. The Soviet Union currently has the capability to place up to 35 RVs on a single missile. This is done by reducing the size and weight of individual RVs. A reduction in RV size and weight causes a concomitant reduction in RV yield (Ref 19).

In a recently released report, the House Armed Services Committee strongly endorsed the Army's LOAD system and urged the Army to expedite its program. The army is continuing its research and development program (Ref 18).

Problem Statement

The problem dealt with in this research is the improvement of MX survivability. Two specific means of improving MX survivability are compared. One is the deployment of LOAD, and the other is increasing the MX shelter hardness.

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Hardness is the measure of a structure's ability to with-
stand increases in pressure beyond normal levels.

Objectives

This research has two major goals. The first goal is
to determine which interceptor strategy provides MX with the
highest expected probability of survival. The second goal
is to compare the effectiveness of increasing MX shelter
hardness to LOAD deployment as methods to improve MX
survivability.

Specific Goals

In order to accomplish these objectives, the following
goals have been established for this research. They are:

    1. Construct a computer simulation which will
       compute MX survivability.

    2. Explore the relationship between interceptor
       yield and circular error probable (CEP) and
       determine their impact on interceptor
       effectiveness.

    3. Examine the effect of RV CEP on probability
       of kill (PK).

Scope

One area of investigation is the effect of fractioni-
zation by the enemy on the effectiveness of LOAD. Fraction-
ization is the process by which the enemy might increase
the number of RVs on a missile by decreasing the yield of
each RV.

The impact on LOAD effectiveness of changes in the
yield and CEP of the interceptors is explored. CEP is an
accuracy measure such that low CEP means more accurate.
Obviously, the most effective interceptor has a large
yield and a low CEP. For reasons other than effectiveness,
a low yield and large CEP are sought. A low yield is desired
so that the amount of nuclear radiation released in the
atmosphere by interceptor detonation is kept at a low level.
A large CEP is desired since the cost of the LOAD system
increases as CEP decreases. However, interceptor yield and
CEP must be at levels which provide the LOAD system with
the ability to improve MX survivability.

A limited number of the parameters for this problem
will be incorporated in the model. The parameters of the
attacking RVs which are included are number targeted on an
MX complex, yield, and CEP. Interceptor parameters included
are number of interceptors per DU, yield, CEP, and strategy
of employment. The MX shelter parameters are the sure-safe
and sure-kill overpressure levels. Sure-safe level is a
measure of a target variable below which the target will

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not be destroyed 98 percent of the time. Sure-kill level is a measure of a target variable above which the target will be destroyed 98 percent of the time. Investigation of the effects of parameters other than these is outside the scope of this research.

Methodology

The system science paradigm was used to develop the model used in this investigation. The system science paradigm is an iterative process of conceptualization, analysis, and computerization (Ref 21:297).

The programming of the model was done in the simulation language Q-GERT. This language was used because it provides for the insertion of FORTRAN subprograms and can readily incorporate probabilistic events (Ref 16).

II. The Model

The System

The modeled system can be divided into three separate subsystems: attack, target, and defense. Each subsystem will be discussed individually.

Attack Subsystem. The attack subsystem is made up of the attacking RVs and the Soviet missiles used to launch these RVs. Since the MX shelters will be hardened structures, the Soviet Union is expected to target the shelters with a counterforce weapon. A counterforce weapon is one which has a low CEP and sufficient yield to destroy hardened targets such as missile silos and shelters. The SS-18 and SS-19 ICBMs are two Soviet missiles which can deliver counterforce weapons. In the near future, the Soviet Union will have 308 SS-18 and 360 SS-19 ICBMs operational. It is believed that, in their most lethal configurations, SS-18s will be able to deliver 10 multiple independently targetable reentry vehicles (MIRVs), each having a CEP of .14 nautical miles (NM) and a yield of 500 kilotons (KT). The SS-19 could be deployed with six MIRVs, each having a CEP of .14 NM and a yield of 550 KT. Thus, the Soviet Union would be able to launch up to 5240 RVs at the MX shelters using the SS-18 and SS-19 ICBMs (Ref 24).

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If the Soviet Union planned to attack MX with RVs delivered by the SS-18, they could target the 200 MX complexes with 3080 RVs. To increase the number of attacking RVs, the Soviet Union could deploy more SS-18s, develop the ability to reload SS-18 silos, and/or increase the number of RVs launched by an SS-18 through the process of fractionization. With the ability to target more than one RV on an MX shelter, fratricide becomes a problem. Fratracide is the destruction of an RV by another RVs detonation. However, many experts believe this problem can be overcome by careful RV timing and targeting (Ref 6:34).

Target Subsystem. The target subsystem will be the 200 MX missiles, the 4600 hardened, horizontal MX shelters, and, if present, the DU and its accompanying components. Since each MX missile will be hidden in one of 23 shelters, one MX missile will represent 23 aimpoints. The distance between shelters should be approximately 7000 feet, and this spacing should prevent the destruction of more than one shelter by an attacking RV (Ref 13:14). Each shelter will be a hardened structure, and the hardness of a shelter will depend on the shelter's design. If a shelter is built with four roof portals to allow Soviet verification in accordance with the Strategic Arms Limitations Talks agreements, the MX shelter's expected hardness to overpressure would be 600 pounds per square inch (psi) (Ref 12). However, if the design of the shelters is changed and the number of

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roof portals is decreased, the MX shelter could be hardened to levels in excess of 1000 psi (Ref 10:58).

If the defensive subsystem is present, its radar network becomes part of the target subsystem. If the DU's radar network is destroyed, the DU loses its ability to defend MX. Several methods have been proposed for the defense of the radar network. These include a non-nuclear defensive missile system, mobile radar units, and deployment of redundant, dispersed units. Although each of these proposals has positive and negative aspects, it is felt that the DU's radar network can be made survivable (Ref 20).

Defensive Subsystem. The defensive subsystem will include the DU's radar network, high-speed nuclear armed interceptor missiles, and a control unit. Part of the radar network, the interceptors, and the control unit will be housed in an MX shelter (Ref 8).

The radar network of the subsystem might have three stages. The first stage would be an early-warning system which would detect incoming RVs aimed at the MX field. The second stage might be an MX complex radar warning system. This stage would detect incoming RVs targeted on the MX complex. If incoming RVs are descending on the complex, the last stage of the radar would begin to function. This radar would track incoming RVs and determine their target.

If the MX or DU shelter was targeted, the control

would determine whether or not its preprogrammed instructions required an interceptor launch. If interceptor launch is required, the DU would leave the shelter, acquire the RV with its radar, launch an interceptor, and return to the shelter. If an interceptor is launched, it would detonate at an altitude of 5000 to 15000 feet and attempt to destroy the RV with a shower of neutrons (Refs 8; 18; 27:1137; 25; 7:60).

System Variables

The system contains three distinct subsystems: attack, target, and defense. Each subsystem presents a set of variables, and each variable has an impact on MX survival. The major variables of each subsystem will be presented below.

The attack subsystem's ability to destroy an MX depends on many factors. The number of RVs attacking a complex, the yield and CEP of each RV, and RV reliability are important factors. Other factors include the RV's ability to survive the effects of an interceptor detonation and the targeting plan of the attack. The targeting plan affects the problem of RV fratracide. RV height of burst has an impact on the effectiveness of the attack. RV reentry angle and speed also have an effect on attack success.

Several target variables have an effect on MX survivability. Shelter hardness is an important factor.

Hardness depends on shelter design, construction quality, and type of soil in which it is built. Other target variables include number of shelters per complex, shelter spacing, weather conditions such as rain or dust storms, and number of MX missiles per complex.

Variables of the defensive system are interceptor yield, CEP, maximum speed, and reliability. Number of interceptors per DU, number of DUs per MX complex, and interceptor strategy are variables. The reliability and accuracy of the radar network are also factors.

The above variables are not independent. They interact with each other to produce the final result which is number of MX missiles remaining operational. Since an attack of this nature has never occurred, there may be unknown side effects such as earthquakes. A nuclear war can be fought only once, so the only reasonable way to estimate the outcome is to create a model which considers those variables which are deemed most important.

Structural Model

The variables chosen in this system were selected because they were identified as being important in reaching the objectives and goals of the thesis. Certain variables were given a preassigned value. Other variables were made parameters of the model. These parameters could then be varied from one model run to the next. The variables are

identified with the appropriate subsystem in the following presentation. Those variables treated as parameters in the model are identified.

The attack subsystem's variables chosen for modeling are RV yield, RV CEP, number of RVs attacking an MX complex, RV height of burst, RV sure-kill and sure-safe neutron fluence levels, and RV reliability. The variables which are model parameters are RV yield, RV CEP, and number of RVs attacking the MX complex.

In modeling the target subsystem, the following variables were incorporated: shelter sure-safe and sure-kill overpressure levels, type of soil in which the shelters are located, number of shelters per complex, and the spacing between shelters. Sure-safe and sure-kill overpressure levels are model parameters.

Defense subsystem variables included in the model are interceptor CEP, interceptor strategy, interceptor yield, number of interceptors per DU, number of DUs per MX complex, radar network reliability and survivability, and interceptor reliability. The parameters selected in the modeling of this system were interceptor yield, number of interceptors per complex, interceptor strategy, and interceptor CEP.

The variables assigned fixed values became part of the boundary conditions of the experiment. The reliabilities of the RVs, interceptors, and radar network have been set to one. The goals of this research do not include an

exploration of the effects of reliability on MX survivability. It is assumed that the radar network will be deployed in such a fashion so that the system will be survivable in a nuclear environment. If the defensive subsystem is present, the number of interceptors per DU is three, the number of DUs per MX complex is one, the number of MX missiles per complex is one, and each MX complex has 23 shelters. These values are based on current Department of Defense plans (Refs 25, 26). The height of burst of incoming RVs has been set at zero. That is, the RV is detonated by surface contact. The reason for this is that a surface burst is required to destroy hardened targets such as MX shelters (Ref 11:93). Sure-kill and sure-safe neutron fluence levels of the RVs have been set at 10^{17} and 10^{13} neutrons per square centimeter (n/cm^2), respectively. Actual levels would depend upon the design and make-up of the attacking RVs. 10^{17} n/cm^2 is a reasonable estimate of the sure-kill fluence of an RV. The sure-safe estimate is based upon the fact that neutron fluence levels for aircraft are approximately the sure-kill fluence levels divided by 10^4 (Ref 3). It is assumed that the MX shelters are built in dry soil or dry soft rock. This is a reasonable assumption because of the arid conditions which prevail in Nevada and Utah. It is assumed that the spacing between shelters is sufficient to prevent the destruction of two shelters by one RV.

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The model provides for the choice of interceptor strategies based on the number of interceptors which will be used to defend the MX missile shelter only. Interceptors not reserved for MX defense can be used to defend either the MX or DU.

Probabilities of Kill

The calculation of probabilities of kill (PKs) of the RVs and interceptors is an important function of the models. To perform these calculations, several assumptions are made.

The products of a nuclear blast detonated by surface contact are wind gusting or dynamic pressure, blast or overpressure, neutrons, gamma rays, thermal radiation, ground motion and a crater. In designing a hardened shelter for a missile, the effects of many of these phenomena can be ignored. If a structure is built flush with the ground, the destructive sideloadings of dynamic pressure can be avoided. Thick, steel reinforced concrete walls provide shielding against gamma rays, neutrons, and thermal radiation (Ref 11). The MX shelter should thus be able to withstand dynamic pressure, thermal radiation, gamma rays, and neutrons and shield the MX and the DU from their harmful effects. The MX shelters will be built with a suspension system which will prevent missile damage by ground motion (Ref 23). The shelter can be hardened to withstand high levels of overpressure, but these levels depend on shelter

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design. However, overpressures in excess of shelter
limitations will crush a shelter. If the shelter is
within the radius of the crater created by the nuclear
detonation, the shelter will be destroyed. Therefore,
if a nuclear detonation can place excessive overpressure
on a shelter or place the shelter within its crater, the
shelter is destroyed. The model routines which calcu-
late the PKs due to cratering and overpressure are
presented in Appendices A and B, respectively.

The detonation of a nuclear-armed interceptor missile
within the atmosphere produces overpressure, dynamic pres-
sure, thermal radiation, gamma rays, and neutrons. An
RV reentering the atmosphere should be able to withstand
the effects of thermal radiation as it is designed to
withstand the high temperatures created by its reentry
into the atmosphere. Overpressure and dynamic pressure
have little effect on incoming RVs because of aerodynamic
design and high reentry speeds (Ref 4). The material of
which the outer shell of the RV is made should provide
shielding from gamma rays (Ref 11:336). However, the
neutrons created by the interceptor's detonation will
destroy the RV if their fluence level is sufficiently high
(Ref 7:1137). Thus the neutrons created by the interceptor's
detonation are the interceptor's kill mechanism.

To use the PK routine presented in Appendix B, the
sure-safe and sure-kill ranges of the RV when subjected

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to neutron fluence must be provided. Their calculation
is presented in Appendix C.

The Simulation

The model simulates the attack of the target and
its defense by use of the simulation language Q-GERT. A
listing of the computer program and the accompanying
Q-GERT network are presented in Appendix D.

In the simulation, the specified number of attacking
RVs is generated. Each group of 23 RVs is targeted on
the 23 MX shelters on a one-to-one basis. Excess RVs are
randomly targeted on the shelters. The MX and the DU, if
present, are randomly assigned to a shelter. The simula-
tion then determines how many RVs are targeted on the MX
and DU shelters. If no RV is aimed at the MX shelter, the
simulation stops, and the MX is not destroyed. If RVs
are aimed at the MX shelter, the simulation continues.
If the defense is present, it attempts to defend the MX
and the DU shelters according to the designated interceptor
strategy. If the DU shelter is destroyed, the MX cannot be
defended. The RVs attack the shelter in a random fashion
with one RV at a time attacking and defended against. The
simulation determines the number of RVs reaching the MX
shelter and calculates the PK of the attack using the
following formula:

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$$PK_A = 1 - (1 - PK_R)^n$$

where

$$PK_A = \text{PK of the attack;}$$

$$PK_R = \text{PK of one RV;}$$

$$n = \text{number of RVs reaching MX shelter.}$$

Using this PK, the simulation then determines whether the
MX is destroyed or not destroyed.

Verification

Verification of the model was accomplished by simulat-
ing two different attacks on a defended MX field and com-
paring the model results with results derived analytically.
Attack one attacked a MX complex with 15 RVs having a yield
of 500 KT and a CEP of 850 feet. This configuration gives
each attacking RV a PK of one. Attack two attacked a MX
complex with 15 RVs having a yield of 500 KT and a CEP of
1400 feet. The PK of one RV is 0.64 in attack two. The
MX complex was defended by one DU with three interceptors.
The parameters of the interceptors are yield and CEP, and
each parameter was tested at four levels. The interceptors
thus had 16 different configurations. The PKs of these

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interceptors are presented in Table I.

TABLE I

Interceptor PKs

Yield (KT)	CEP (feet)			
	250	400	600	900
5	.65	.50	.37	.27
10	.75	.59	.45	.33
20	.85	.70	.55	.40
50	.94	.85	.71	.54

The formulation of the attacks and the MX defenses
permits the analytic computation of the probabilities of
kill (PKs) of the MX. These are presented in Tables II
and III. The model was run 200 times for each set of
parameters with shelter sure-safe and sure-kill over-
pressure levels set at 250 and 750 psi, respectively.
These results are presented in Tables IV and V.

The output of one run of the model is MX destroyed
or not destroyed. This type of output is a Bernoulli
trial and the results of multiple Bernoulli trials can
be characterized by the binomial distribution (Ref 22:191).
The binomial distribution can be approximated by the
normal distribution if n , the number of runs, is suffi-
ciently large, and p , the probability of MX destruction,

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TABLE II

Analytic Results of Attack One

Attack PK with Given Interceptor CEP and Yield

Yield (KT)	CEP (feet)			
	250	400	600	900
5	.23	.33	.41	.48
10	.16	.27	.36	.44
20	.10	.20	.29	.39
50	.04	.10	.19	.30

TABLE III

Analytic Results of Attack Two

Attack PK with Given Interceptor Yield and CEP

Yield (KT)	CEP (feet)			
	250	400	600	900
5	.15	.21	.26	.30
10	.11	.17	.23	.28
20	.07	.13	.19	.25
50	.03	.06	.12	.19

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TABLE V

Simulation Results for Attack One

Attack PK with Given Interceptor Yield and CEP

	CEP			
Yield (KT)	250	400	600	900
5	.31	.38	.48	.56
10	.18	.30	.41	.50
20	.14	.23	.30	.43
50	.04	.14	.22	.30

TABLE V

Simulation Results for Attack Two

Attack PK with Given Interceptor Yield and CEP

Yield (KT)	250	400	600	900
5	.19	.21	.26	.32
10	.10	.16	.23	.29
20	.07	.13	.16	.24
50	.04	.07	.12	.16

is close to one-half. In general, the approximation is good if $np > 5$ when $p \leq 0.5$ or $n(1 - p) > 5$ when $p > 0.5$ (Ref 14:181-182). This is true of the model's output when $n = 200$, so the following equation can be used to define a confidence interval:

$$d^2 = Z^2 \alpha/2 \: / \: (4n)$$

where d is the interval about the true mean, $Z \: \alpha/2$ is the two-tailed standardized normal statistic for the confidence interval, and n is the number of runs. For a 95 percent confidence interval $z_{\alpha/2} = 1.96$ and given 200 runs, $d \cong \pm.07$. Thus there is confidence that 95 percent of the results of the model will be within $\pm$ seven percent of the true mean (Ref 22:191-192).

To test the null hypothesis that the PK of the MX computed by the model equals those found analytically, a test of hypothesis of binomial parameters was used. This test is based on the normal approximation of the binomial distribution. The hypotheses are:

$$H_0 : p = p_0$$

$$H_1 : p \neq p_0 \quad .$$

The test is:

$$Z_o = \begin{cases} \dfrac{x + 0.5 - np_o}{\sqrt{np_o(1 - p_o)}} & \text{if } x \le np_o \\[4mm] \dfrac{(x - 0.5) - np_o}{\sqrt{np_o(1 - p_o)}} & \text{if } x > np_o \end{cases}$$

and fail to reject the null hypothesis if

$$-Z_{\alpha/2} \le Z_o \le Z_{\alpha/2}$$

where

x = number of missiles destroyed in n runs

p = PK computed by the model

p_o = the analytically achieved PK (Ref 14:283-284).

α is the probability that the null hypothesis is rejected when it is true and is set at the .05 level. Using this criteria, the null hypothesis can be rejected only twice in 32 tests made between model results and analytic results. $\frac{2}{32}$ = .06 $\simeq$ 5% . Therefore, the model results failed to compare with analytic results in 6.25 percent of the tests.

This is expected for a 95 percent confidence interval, and thus the model functions properly.

Validation

The system the model portrays has not been built. It is impossible to compare the behavior of the model with the behavior of the real system. An attempt has been made to include variables of the real system which are anticipated to have a significant effect on the probability of MX destruction. Here, projected real-world values were used in the simulation, and reasonable assumptions were used to model those areas where data was not available. The model was constructed so that the results achieved would provide information necessary for meeting the objectives of the thesis. Within the stated limitations, the model is valid.

III. The Analysis

Research Design

The model is designed to provide estimates of the probability of kill of the MX when attacked by enemy RVs and defended by interceptor missiles contained on a DU. The model allows an investigation of the effects of different interceptor strategies and different levels of shelter hardness on the PK of the MX.

The effective use of the model hinges on determining the number of runs of the model required to ensure a desired confidence interval. A method which compares the results of the model must be developed, and the levels of the model parameters which will be explored must be chosen.

Number of Runs. Since the output of one run of the model is a Bernoulli trial, the model results can be characterized by the binomial distribution. The binomial distribution can be approximated by the normal distribution for reasonable large sample sizes. It can be shown that

$$n = Z^2_{\alpha/2}/(4d^2)$$

where n is the number of model runs, d is the desired interval about the true mean, and $Z_{\alpha/2}$ is the standardized normal statistic for the probability sought (Ref 22:191-192).

It is desired that the model results differ from the true PK by no more than four percent with a confidence level of 95 percent which implies $Z_{\alpha/2}$ equals 1.96 . Given these inputs, the number of model runs required is 600.

Statistical Test. The results produced by the model must be compared to determine which strategy produces the lowest MX PK. A test on the means of normal populations with variances unknown and not assumed equal is used to make the comparisons. The use of this test takes advantage of the fact that, for a large sample size such as 600, a binomial population is closely approximated by a normal population. The test statistic used is:

$$t_o = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$$

where

$\bar{x}_1$ and $\bar{x}_2$ = sample means

S_1^2 and S_2^2 = sample variances

n_1 and n_2 = sample sizes.

If P is the number of samples in which the MX is destroyed, the following equations can be used to compute the sample means and variances:

$$\bar{x} = P/n$$

$$S^2 = n(P/n)((1 - P)/n)/(n - 1)$$

where n is the number of samples. The test statistic t_0 is then compared to $t_{\alpha/2,\gamma}$ in the case of equality. The hypotheses are:

$$H_0 : u_1 = u_2$$

$$H_1 : u_1 \neq u_2$$

and the null hypothesis is rejected if

$$-t_0 < t_{\alpha/2,\gamma} < t_1 \quad .$$

$t_{\alpha/2,\gamma}$ is the two tailed t statistic with given degrees of freedom, γ , and α is the probability of rejecting the null hypothesis when it is true. The α error is 0.05. γ , the number of degrees of freedom, is computed by the following equation:

$$\gamma = \frac{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)^2}{\dfrac{(S_1/n_1)^2}{n_1+1} + \dfrac{(S_2^2/n_2)^2}{n_2+1}} - 2 \quad .$$

The number of degrees of freedom for sample sizes of 600 can never be less than 599. At 599 degrees of freedom, the t distribution degenerates to the normal distribution. Thus $t_{\alpha/2,\gamma}$ equals 1.96. The one-tailed test differs slightly. For the case of less than, the null hypothesis, $H_0 : u_1 \leq u_2$, is rejected if $t_0 > t_{\alpha,\gamma}$, and for the case of greater than, the null hypothesis $H_0 : u_1 \geq u_2$, is rejected if $t_0 < -t_{\alpha,\gamma}$ where $t_{\alpha,\gamma}$ equals 1.645 (Ref 14:263-267).

Selection of Model Parameter Levels

The parameters of the attack are number of RVs attacking the MX complex, RV yield, and RV CEP. Three levels of attack were used to test the effectiveness of the defense and shelter hardness. The missile assumed to be launching the RVs is the SS-18. Fractionization was used to increase the number of RVs delivered by each enemy missile. Through this process, the number of RVs delivered by each SS-18 was increased from 10 to 20 to 30 RVs to yield the three levels of attack. Assuming 300 SS-18s were dedicated to the destruction of the MX missiles,

the number of RVs attacking a MX complex would be 15, 30, or 45 RVs. When fractionization is used to increase the number of attacking RVs, the yield of each of the RVs must be decreased because total missile payload is constant. A SS-18 can deliver 10 RVs with yields of 500 KT each (Ref 24:70). The yield of a RV when the SS-18 launches 20 or 30 RVs was assumed to be 250 and 125 KT, respectively.

Thus, three enemy strategies are simulated. First, a low density attack which assumes 10 RVs per missile, each with a yield of 500 KT. Second, a medium density attack which assumes 20 RVs per missile, each with a yield of 250 KT. Finally, a high density attack is considered which assumes 30 RVs per missile, each with a yield of 125 KT.

The CEP of an attacking RV is critical in determining the effectiveness of a RV. Accurate estimates of the accuracy of enemy missiles are classified. The problem of making these estimates is summarized by Feld and Tsipis (Ref 10:54).

> "CEP is defined as the radius of the circle around a target within which half the warheads aimed at the target can be expected to land. The rated CEP of any missile system is established in test flights, and it may or may not be duplicated in actual military operations. Furthermore, this measure of accuracy does not take into account the probability of systematic aiming errors. The CEP value for Russian missiles is presumably determined by U.S. intelligence experts from information gathered during Russian weapons tests by observing the launching of each missile and the return of its warhead (or warheads) to the surface. The U.S. Government, however, does not announce the results

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of its intelligence-gathering activities. As
a result the CEP values of Russian ICBMs cited
in unofficial public statements... may be
neither the values measured directly by the
Russians nor the values determined indirectly
by the Americans. Since accuracy is the most
important determinant of a missile's destructive-
ness against a silo, publicly quoted estimates
of the CEP values of Russian missiles are likely
to be subject to powerful political pressures
generated by highly motivated interests."

In 1978, the SS-18 ICBM was reported to have achieved

accuracies of .1 NM (Ref 17). In 1980, the CEP of

.14 NM was claimed for the SS-18 (Ref 24). There is

evidence, however, that the guidance packages which

achieved these low CEPs are intended for the new genera-

tion of ICBMs still in development (Ref AA:15). The

CEP of the SS-18 is within the range of .12 to .25 NM

(Ref 10:54). For the purpose of this research effort,

the CEP of the Soviet SS-18 ICBM will be assumed to be

.23 NM or 1400 feet. This CEP gives the SS-18 a hard

target kill capability. The PKs of single RVs with the

specified yields attacking MX shelters with sure-safe and

sure-kill levels of 250 and 750 psi, respectively, can be

seen in Tables VI, VII, and VIII. As CEP increases, the

PKs of the RV drops quite dramatically. Thus CEP is

critical in the determination of RV lethality, and a

CEP of 1400 feet appears to be reasonable.

Target parameters are shelter sure-safe and sure-kill

overpressure levels. If a DU is deployed, the sure-safe

level is 250 psi, and the sure-kill level is 750 psi.

TABLE VI

PK of 500 KT RV Against MX Shelter

CEP (Feet)	PK of One RV
500	1.
600	1.
700	.999998
800	.99992
900	.997522
1000	.971077
1100	.88152
1200	.758423
1300	.673958
1400	.637124
1500	.622592
1750	.60421
2000	.583191
2250	.506442
2500	.361856
2750	.255757
3000	.215706

TABLE VII

PK of 250 KT RV Against MX Shelter

CEP (Feet)	PK of One RV
500	1
600	.999985
700	.998599
800	.966807
900	.840595
1000	.703604
1100	.643212
1200	.624145
1300	.614017
1400	.60572
1500	.597088
1750	.529103
2000	.353
2250	.239186
2500	.208875
2750	.197688
3000	.189461

TABLE VIII

PK of 125 KT RV Against MX Shelter

CEP (Feet)	PK of One RV
500	.999938
600	.989792
700	.870304
800	.69885
900	.637437
1000	.620385
1100	.609299
1200	.598113
1300	.576071
1400	.523587
1500	.437327
1750	.253911
2000	.210214
2250	.196923
2500	.18719
2750	.179251
3000	.172635

When the MX is not defended, the following hardness levels
were tested to determine their effect on MX survival:

Sure-Safe (psi)	Sure-Kill (psi)
250	750
500	1000
750	1250
1000	1500
1250	1750

The hardness of the shelters will depend on their final
design and construction. Since they have not yet been
built, their hardness levels are subject to change.
Although a horizontal shelter can be hardened to levels
exceeding 1000 psi (Ref 10:58), 1750 psi is chosen as
the maximum hardness level because of the large surface
area of a horizontal shelter. The 500 psi difference
between sure-safe and sure-kill levels is selected as a
reasonable guess.

The parameters of the defense are interceptor yield,
interceptor CEP, interceptor strategy, and number of
interceptors per MX complex. The number of interceptors
is set at three because, under current planning, three
interceptors will be deployed with the DU of an MX complex
(Ref 25:26). However, if the MX is undefended, the number
of interceptors deployed can be set to zero in the model.

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In selecting the interceptor yields and CEPs which would
be used as model inputs, the following criteria were
established. A LOAD interceptor will be half the size of
the Spring missile (Ref 8). Its size will limit the yield
of its warhead. The interceptor will detonate at a low
altitude and release nuclear radiation into the atmosphere
of the United States. Because of the interceptor's small
size and the deisre to keep released radiation at a mini-
mum, the yield of the interceptors does not exceed 20 KT
in the model. A lower limit on the PK of the interceptor
is desired. It is assumed that an interceptor with a PK
less than 40 percent will not be deployed. Given this
lower limit and a maximum yield of 20 KT, it was found by
using the model in Appendix B that a CEP of 900 feet pro-
duced a PK of 40 percent. Therefore, the CEP of an inter-
ceptor can be assumed to be 900 feet or less. Given these
limits on CEP and yield, three CEPs (300, 600 and 900 feet)
and three yields (5, 10 and 20 KT) were selected for inves-
tigation. The interceptor PKs for the nine combinations of
CEP and yield are shown in Table IX. Interceptors with a
yield of 5 KT and CEPs of 600 and 900 feet and the interceptor
with a yield of 10 KT and a CEP of 900 feet are eliminated
from further consideration because their PKs are less than
40 percent. Thus six different interceptor configurations
are included in the modeling process.

Three strategies for interceptor usage are examined
to determine which is most effective in defending MX.

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TABLE IX

Interceptor PKs

Yield (KT)	300	600	900
5	.59	.37	.27
10	.69	.45	.33
20	.79	.55	.40

Strategy One allows the DU to launch the first two inter-
ceptors at RVs aimed at either the MX or DU shelter. The
remaining interceptor will be used for MX defense only. If
an RV is attacking the DU shelter and only one interceptor
remains, the interceptor will not be launched, and the DU
will be subjected to an RV detonation which may or may not
destroy the DU.

Strategy Two permits the launch of the first interceptor
at an RV aimed at either the DU or the MX shelter. The two
remaining interceptors may only be launched at RVs targeted
on the MX shelter.

Strategy Three does not allow the DU to defend itself.
RVs aimed at the DU shelter will not be intercepted. This
strategy confines the use of the interceptor to MX defense
only.

Model Runs

For the defended system, the model was run 54 times for all combinations of the three levels of attack defended by six interceptor configurations using three defensive strategies. For the case of the undefended system, 15 runs were made for three levels of attack and five different shelter hardness levels.

Results

The results of the simulation are in the form of MX probability of kill (PK). For each set of inputs, the outputs are presented in tabular form in the following manner:

$$X(Y)$$

where X is the MX PK and Y is the number of MX missiles killed in 600 trials. The results are shown in Tables X through XIII.

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TABLE X

MX PK From Low Density Attack

DU Strategy 1

Yield (KT)	CEP (Feet)		
	300	600	900
5	20(120)		
10	18(108)	25(150)	
20	13(78)	22(133)	26(153)

DU Strategy 2

Yield (KT)	CEP (Feet)		
	300	600	900
5	30(120)		
10	18(109)	25(150)	
20	13(78)	22(133)	26(153)

DU Strategy 3

Yield (KT)	CEP (Feet)		
	300	600	900
5	22(133)		
10	19(115)	27(160)	
20	17(102)	24(141)	28(167)

TABLE XI

MX PK From Medium Density Attack

DU Strategy 1

Yield (KT)	CEP (Feet)		
	300	600	900
5	36(216)		
10	31(188)	45(268)	
20	24(144)	39(232)	49(292)

DU Strategy 2

Yield (KT)	CEP (Feet)		
	300	600	900
5	40(242)		
10	32(190)	50(301)	
20	23(136)	40(239)	52(311)

DU Strategy 3

Yield (KT)	CEP (Feet)		
	300	600	900
5	43(258)		
10	39(233)	49(292)	
20	34(204)	45(267)	52(313)

TABLE XII

MX PK From High Density Attack

DU Strategy 1

Yield (KT)	CEP (Feet)		
	300	600	900
5	53(317)		
10	45(269)	53(318)	
20	31(187)	59(331)	59(356)

DU Strategy 2

Yield (KT)	CEP (Feet)		
	300	600	900
5	48(286)		
10	45(272)	61(365)	
20	42(253)	49(295)	60(357)

DU Strategy 3

Yield (KT)	CEP (Feet)		
	300	600	900
5	53(316)		
10	48(288)	60(357)	
20	43(259)	55(328)	61(365)

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TABLE XIII

PKs of Attacks on Undefended MX Complex

Low Density Attack

Sure-Kill (psi)	PK
750	41(245)
1000	41(245)
1250	40(241)
1500	40(241)
1750	40(241)

Medium Density Attack

Sure-Kill (psi)	PK
750	68(406)
1000	67(404)
1250	59(354)
1500	39(231)
1750	36(218)

High Density Attack

Sure-Kill (psi)	PK
750	76(453)
1000	47(279)
1250	43(260)
1500	43(260)
1750	43(260)

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Analysis

The statistical test presented earlier was used to compare the results of the three strategies. In the following tables, "EQ" means the null hypothesis of equality could not be rejected at .05 level. "LT" means the hypothesis of less than could not be rejected at the .05 level. "GT" means the hypothesis of greater than could not be rejected at the .05 level.

Comparisons Between Defensive Strategies. The comparisons among the three DU strategies when defending MX against each of the three attacks produced the results shown in Tables XIV, XV, and VXI. The results from Tables XIV, XV, and XVI are summarized in Table XVII. In Table XVII, "EQ" means the strategies produced statistically equal PKs. "LT" means the PKs of the first strategy are statistically less than the PKs of the second strategy. The "LT" thus implies the first strategy is a more effective strategy than the second. Table XVII shows Strategy Three is the least effective strategy. There is doubt as to which of the two remaining strategies is most effective. Strategy One and Strategy Two are equally effective against the low and medium density attacks. Against the high density attack, two interceptor configurations are more effective when using Strategy One and one configuration is more effective when using Strategy Two. The other three configurations are equally effective using either strategy. Rather than guess at which strategy

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TABLE XIV

Comparison of DU Strategies When MX is Subjected to a Low Density Attack

Strategy One vs Strategy Two

Yield (KT)	CEP (Feet)		
	300	600	900
5	EQ		
10	EQ	EQ	
20	EQ	EQ	EQ

Strategy One vs Strategy Three

Yield (KT)	CEP (Feet)		
	300	600	900
5	EQ		
10	EQ	EQ	
20	EQ	EQ	EQ

Strategy Two vs Strategy Three

Yield (KT)	CEP (Feet)		
	300	600	900
5	EQ		
10	EQ	EQ	
20	EQ	EQ	EQ

TABLE XV

Comparison of DU Strategies When MX
is Subjected to a Medium Density Attack

Strategy One vs Strategy Two

	CEP (Feet)		
Yield (KT)	300	600	900
5	EQ		
10	EQ	EQ	
20	EQ	EQ	EQ

Strategy One vs Strategy Three

	CEP (Feet)		
Yield (KT)	300	600	900
5	LT		
10	LT	EQ	
20	LT	LT	EQ

Strategy Two vs Strategy Three

	CEP (Feet)		
Yield (KT)	300	600	900
5	EQ		
10	LT	EQ	
20	LT	EQ	EQ

TABLE XVI

Comparison of DU Strategies When MX is Subjected to a High Density Attack

Strategy One vs Strategy Two

	CEP (Feet)		
Yield (KT)	300	600	900
5	EQ		
10	EQ	LT	
20	LT	GT	EQ

Strategy One vs Strategy Three

	CEP (Feet)		
Yield (KT)	300	600	900
5	EQ		
10	EQ	LT	
20	LT	EQ	EQ

Strategy Two vs Strategy Three

	CEP (Feet)		
Yield (KT)	300	600	900
5	EQ		
10	EQ	EQ	
20	EQ	EQ	EQ

TABLE XVII

Comparison of DU Strategies by Attack

Low Density Attack

Strategy One	EQ	Strategy Two
Strategy One	EQ	Strategy Three
Strategy Two	EQ	Strategy Three

Medium Density Attack

Strategy One	EQ	Strategy Two
Strategy One	LT	Strategy Three
Strategy Two	LT	Strategy Three

High Density Attack

Strategy One	??	Strategy Two
Strategy One	LT	Strategy Three
Strategy Two	EQ	Strategy Three

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is more effective in defending MX, additional runs of the model were made with the number of trials increased to 2400. This yields a 95 percent confidence interval about the true mean of plus or minus two percent. Table XVIII presents the results for the three interceptor configurations which gave conflicting indications. Table XVIII indicates Strategy One is more effective than Strategy Two in defending MX for one interceptor configuration. These results indicate that Strategy One, that of using the first two interceptors to defend either the MX or the DU while reserving the last interceptor for MX defense only, is the most effective of the DU strategies.

Comparisons of Shelter Hardness Levels. The effects of increasing shelter hardness were significant decreases in MX PK for some hardness increases and no significant decreases in MX PK for other increases. Increasing shelter hardness had no statistically significant impact on the PKs of the low density attack. When subjected to the medium density attack, MX survival was significantly improved by increasing sure-kill levels from 1000 to 1250 psi and again by hardness increases from 1250 psi to 1500 psi. A marked improvement in MX survival was achieved for the high density attack by increasing the shelter sure-kill level from 750 to 1000 psi. All other increases had no statistically significant effects on MX survivability.

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TABLE XVIII

Strategy One vs Strategy Two

when Subjected to a High Density Attack (2400 Trials)

| Interceptor | | PK | PK | Strategy One vs Strategy Two |
Yield (KT)	CEP (Feet)	Strategy One	Strategy Two	
10	600	56(1347)	58(1398)	EQ
20	300	35(846)	39(931)	LT
20	600	52(1251)	53(1267)	EQ

Comparison Between LOAD Deployment and Increased
Shelter Hardness. A comparison can be made between the
effectiveness of increasing the hardness of undefended
shelters and deploying LOAD. Since DU Strategy One has
already been shown to be the most effective, only that
strategy was considered. Increasing shelter hardness does
not improve MX survivability when subjected to the low
density attack. Therefore, the low density attack is not
included in the comparisons. The results of these compari-
sons against medium and high density attacks are shown in
Tables XIX and XX. In Tables XIX and XX, "EQ" means the
given interceptor and shelter hardness produced statisti-
cally equivalent levels of MX destruction. "LT" means
the interceptor resulted in a statistically significant
lower MX PK than did the shelter's level of hardness. "GT"
means the interceptor resulted in a statistically signifi-
cant higher level of MX destruction than did the shelter
hardness level. If the interceptor or hardness level pro-
duces a statistically significant lower value for MX destruc-
tion, the system is more effective in improving MX surviva-
bility. Deploying LOAD is more effective than increasing
shelter hardness in improving MX survivability when an MX
complex is subjected to a low density attack. Against a
medium density attack, LOAD deployment is more effective.
Shelter hardness is more effective only at hardness levels
above 1500 psi and when compared to interceptors with CEP

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TABLE XIX

DU With Strategy One vs Increased Hardness:

Medium Density Attack

Interceptor		Sure-Kill Levels (psi)				
Yield (KT),	CEP (Feet)	750	1000	1250	1500	1750
5	300	LT	LT	LT	EQ	EQ
10	300	LT	LT	LT	LT	EQ
10	600	LT	LT	LT	GT	GT
20	300	LT	LT	LT	LT	LT
20	600	LT	LT	LT	EQ	EQ
20	900	LT	LT	LT	GT	GT

TABLE XX

DU With Strategy One vs Increased Hardness:

High Density Attack

		Sure-Kill Levels (psi)				
Yield (KT),	CEP (Feet)	750	1000	1250	1500	1750
5	300	LT	GT	GT	GT	GT
10	300	LT	EQ	EQ	EQ	EQ
10	600	LT	GT	GT	GT	GT
20	300	LT	LT	LT	LT	LT
20	600	LT	GT	GT	GT	GT
20	900	LT	GT	GT	GT	GT

(feet)/yield (KT) of 900/20 or 600/10. To combat a high density attack, shelter sure-kill levels above 1000 psi produced a higher expected level of MX survivability. This is true for all comparisons except those with interceptor CEP (feet)/yield (KT) of 300/10 or 300/20. The 300/10 interceptor is equally as effective as sure-kill hardness levels above 1000 psi, while the 300/20 interceptor is more effective than sure-kill hardness levels above 1000 psi.

If a low or medium density attack is expected, then deploying LOAD with Strategy One provides the hgihest expected MX survivability. To combat a high density attack, either shelter sure-kill levels should be 1000 psi or higher, or the interceptors should have relatively high yield and low CEP and be deployed with Strategy One.

IV. Conclusions and Recommendations

Conclusions

Of the three strategies investigated, Strategy One is the
most effective. Strategy Two is almost as effective as Strategy
One and is slightly better than Strategy Three. However,
Strategy One is much better than Strategy Three.

Increases in shelter hardness did not improve MX surviva-
bility of attacks by high yield RVs. As the yield of attacking
RVs decreases and their number increases, increasing shelter
hardness becomes a more effective means of improving MX sur-
vivability. This was quite evident for the attack of an MX
complex by 45 RVs with yields of 125 KT and CEPs of 1400 feet.

CEP is critical in determining the effectiveness of a
weapon. The values attributed to both the enemy RVs and the
interceptors are very important. As RV CEP decreases, MX
becomes less survivable. Lower interceptor CEPs cause dramatic
improvements in interceptor PKs. A decrease of 50 feet in
interceptor CEP from 300 feet to 250 feet increases an inter-
ceptor's PK by five percent.

Against low and medium density attacks, the deployment of
LOAD is a more effective means of improving MX survivability
than increases in shelter hardness. However, against high
density, low yield attacks, increasing shelter hardness is more

effective than LOAD deployment unless LOAD has low CEP, high yield interceptors. Even then, hardness improvement could be more attractive if the release of nuclear radiation in the atmosphere by a LOAD interceptor is an untenable option.

Recommendations

The revived importance of and interest in BMD makes future efforts in this area important. A classified approach could prove most useful as model inputs could be made more realistic.

Several areas of the model could be enhanced. The RV-interceptor engagement could be treated more realistically and the treatment of the nuclear effects could be made more rigorous and precise. Other possible interceptor strategies might be included in the model. For example, two interceptors might be launched against a single attacking RV. The possibilities of deploying more than one DU or MX per complex could be investigated, as could the possibility of building more MX shelters. As the LOAD system is developed and refined, the scope and needs for future research will be increased.

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APPENDICES

APPENDIX A

Probability of Kill Due to Cratering

A nuclear contact surface burst creates a crater. Any object within the apparent radius (R_a) of the crater will be destroyed. To calculate the apparent radius of the crater, the following equation is used:

$$R_a = R_s (Y^{.3})$$

where Y is the yield of the weapon in KT and R_s is the apparent radius of a crater created by a one KT weapon. For a contact surface burst in dry soil, R_s is 61 feet (Ref 11:254-255). The probability of kill (PK) of cratering is defined by the circular normal function and can be found using the following equation:

$$P_K = \int_{-R_a}^{R_a} \frac{1}{\sqrt{2\pi}\, \sigma}\ e^{-\frac{1}{2}\left(\frac{Y}{\sigma}\right)^2} dy$$

where

$$\sigma = CEP/\sqrt{2 \ln 2}\quad .$$

Letting $z = Y/\sigma \rightarrow dy = \sigma\, dz$ and substituting in the above equation gives

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$$PK = \int_{-R_{a/\sigma}}^{R_{a/\sigma}} \frac{1}{\sqrt{2\pi}\ \sigma}\ e^{-\frac{1}{2}z^2}\ dz$$

This implies:

$$PK = \int_{-R_{a/\sigma}}^{R_{a/\sigma}} \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}z^2}\ dz$$

$$= 2 \int_0^{R_{a/\sigma}} \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}z^2}\ dz$$

$$= 2\ [\Phi\ (R_{a/\sigma} - .5)]$$

where Φ represents the cumulative normal distribution (Ref 3). The approximate value of $\Phi(x)$ is found using the following equation:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}z^2}\ dz$$

$$\cong\ 1 - [1/(2[1+.196854x + .115194x^2$$

$$+ .000344x^3 + .019527x^4]^4)]$$

where $x \geq 0$ (Ref 1:932).

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APPENDIX B

PROBABILITY OF KILL ROUTINE

To calculate the probability of kill (PK) of the
shelters due to overpressure, a routine known as the ten
cell model is used. The ten cell model requires the sure-
safe and sure-kill ranges of the shelter. The sure-safe
range is the range at which survival is expected 98 percent
of the time, and sure-kill range is the range at which
destruction is expected 98 percent of the time. In the
model, these ranges must be calculated using the sure-safe
and sure-kill overpressure levels of the shelters. In
computing the PK of an interceptor against an RV, the sure-
safe and sure-kill ranges were calculated separately and
entered in the model as data.

The overpressure created by a contact surface burst is
similar to that of a "free-air" burst with twice the yield.
The graph of peak overpressure (psi) vs distance from burst
in feet for a "free-air" burst of a one KT device is pre-
sented in Figure 3 (Ref 11:91, 109). Reading from the
graph yields the following set of points:

<div align="center">

(feet, psi)

(100 , 2000)

(135, 1000)

(175, 500)

</div>

$$(225, \ 200)$$
$$(300, \ 100)$$
$$(400, \ \ 50)$$
$$(600, \ \ 20)$$
$$(800, \ \ 10)$$

An equation representing this graph is required. To find an approximation of this graph, the following steps were taken:

1. $\ln 850 \cong 6.75$

2. Postulate that equation is of form

$$x(\text{distance}) = e^{(6.75)/(\text{psi}/10)^z}$$

and solve for z.

3. Find z when x = 100 feet and psi = 2000

$$100 = e^{6.75/(2000/10)^z}$$

$$\ln(100) = 6.75 / (200)^z$$

$$(200)^z = 6.75 / (4.61)$$

$$(200)^z = 1.47$$

$$z \ \ln(200) = \ln 1.47$$

$$z \cong .072$$
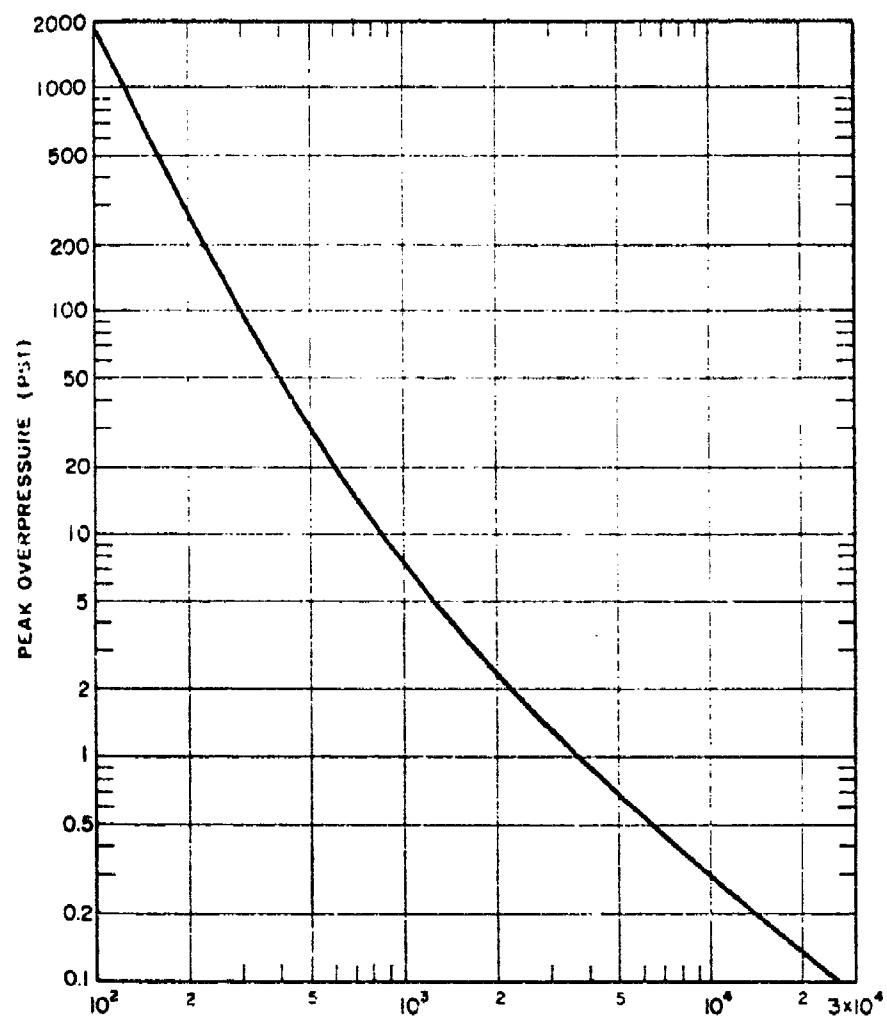
Figure 3. Peak Overpressure from a 1-Kiloton Free Air Burst for Sea-Level Ambient Conditions (Ref 11:109)

4. Therefore

$$x = e^{6.75/(psi/10)^{.072}}$$

so

$$x = e^{7.967/(psi)^{.072}}$$

5. Check the difference between the values read
    from the graph and the values provided by the
    equation.

TABLE XXI

Graph vs Equation

PSI	x(graph)	x(equation)	difference
2000	100	100	0
1000	135	127	8
500	175	162	13
200	225	230	5
100	300	304	4
50	400	408	8
20	600	614	14
10	850	854	4

The equation provides an approximation which is within
7.5 percent of the graph.

The equation developed above provides the scaled range
for a one KT weapon. The range (R) is found using the
following equation:

$$R = R_s (Y^{1/3})$$

where R_s is the scaled range for a one KT weapon and Y
is the weapon yield (Ref 11:108). Thus, the sure-safe and
sure-kill ranges may be found using the following equation:

$$R = \exp(7.967/(psi)^{.072})(2 \ RV \ KT)^{1/3}$$

where psi is the sure-safe or sure-kill psi levels of the
shelters and RV KT is the yield in KT of the attacking
RVs.

The ten cell model is a procedure for calculating the
PK of a given weapon against a designated target. The model
requires the sure-safe and sure-kill ranges of the target,
the CEP of the weapon, and the distance of the target from
designated ground zero (DGZ). DGZ is the point at which
the weapon is aimed. The ten cell model places 10 cells of
equal probability of hit (PH) around the DGZ. That is, the
weapon has an equal probability of impacting in each cell.
Figure 4 shows these cells.

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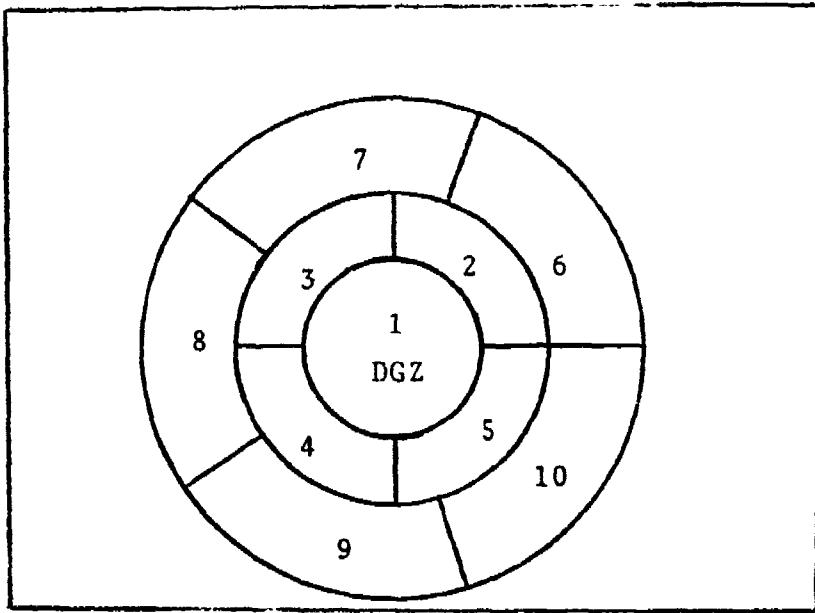
Figure 4.   Ten Cells of Equal PH

Thus, the weapon would have a ten percent chance of impacting in each cell.   It should be noted that distance from DGZ is infinity for the outermost circle of the model.   The case where a target is a given distance from the DGZ and a weapon impacts at some third point is depicted in Figure 5. The PK of a target at a distance x from DGZ is a function of the PH at a point described by ρ and ϕ and the probability of damage (PD) of the target at a distance r from the point of impact.   Thus, the following equation describes the situation:

$$PK(x) = \int_0^2 \int_0^\infty PH(\rho, \phi) \, PD(r) \, r \, dr \, d\phi \quad .$$
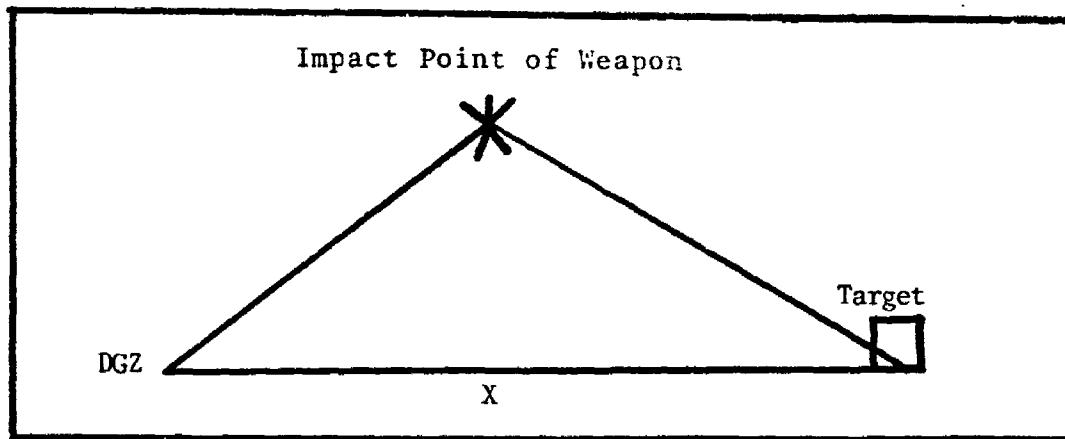
Figure 5. Weapon Impact

Placing the ten cell model over Figure 5 and treating each cell as a discrete impact point produces the following equation:

$$PK(x) = \sum_{i=1}^{N_T} PH(\rho_i, \phi_i) \, \Delta A_i \, PD(r_i)$$

where the variables are defined as follows:

r_i = distance from target to the center of cell i ;

ρ_i = distance from DGZ to outer circle of cell i ;

ϕ_i = angle of cell in relation to DGZ ;

ΔA_i = area of cell i ;

N_T = number of cells in model .

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Other variables of the model are:

$\langle \rho_i \rangle$ = distance from DGZ to probabalistic center of cell i ;

$\langle \phi_i \rangle$ = angle at which probabalistic center of cell i is located;

n_i = number of cells in ring i ;

N_i = number of cells in ring i plus all cells inside this ring.

The following figure will illustrate the above variables. For the ten cell model, the following values can be assigned:

$$N_T = 10 \qquad N_1 = 1 \qquad N_2 = 5 \qquad N_3 = 10$$

$$n_1 = 1 \qquad n_2 = 4 \qquad n_3 = 5 \qquad \rho_3 = \infty \; .$$

Recalling the equation for PK,

$$PK(x) = \sum_{i=1}^{N_T} PH(\rho_i, \phi_i) \, \Delta A_i \, PD \, (r_i) \quad ,$$

$PH(\rho_i, \phi_i) \, \Delta A_i = \frac{1}{N_T}$ for each i since the model is constructed so that an attacking weapon has an equal probability of hitting within each cell. Thus

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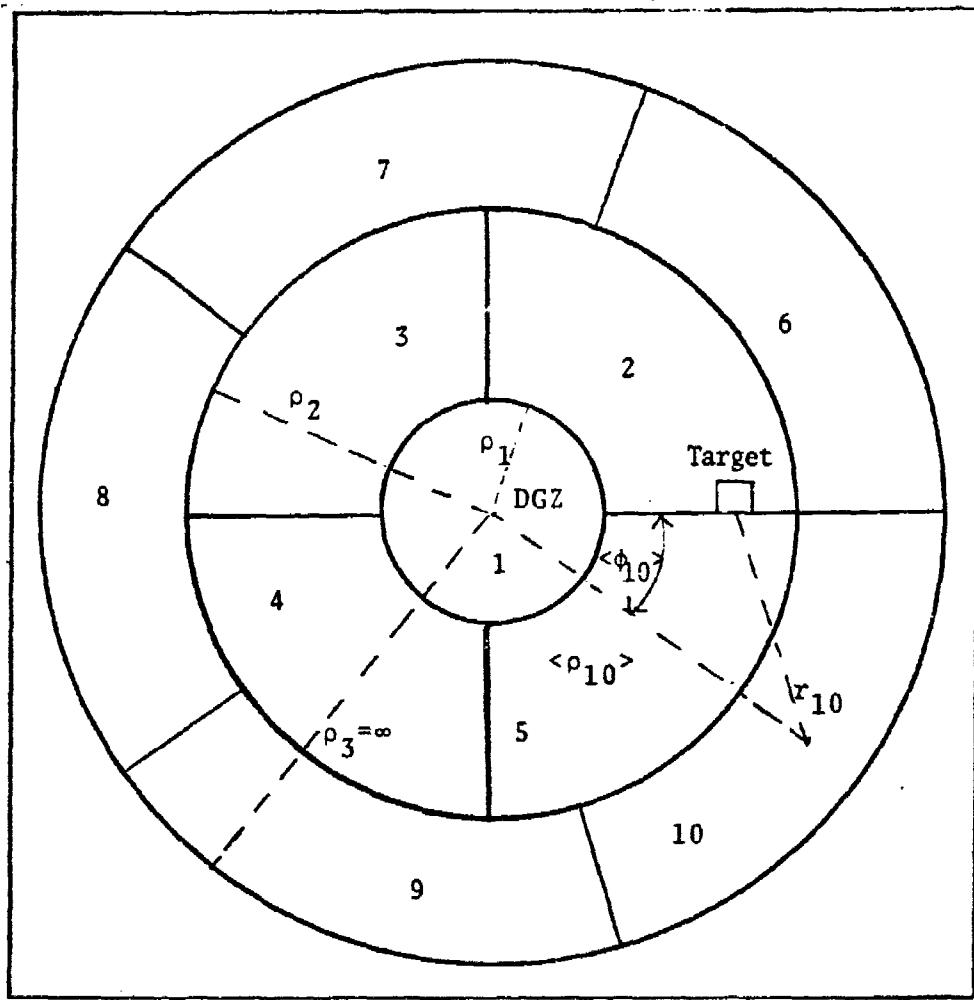
Figure 6.   Ten Cell Model on a Target

$$PK(x) = \frac{1}{N_T} \sum_{i=1}^{N_T} PD(r_i) \quad . \quad \text{From the law of cosines,}$$

$$r_i^2 = x^2 + <\rho_i>^2 - 2x<\rho_i> \cos<\phi_i> \tag{1}$$

To solve for $<\rho_i>$, ρ_i must be found. Since each cell of model represents an equal probability of hit and since

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the probability of hit is distributed according to the circular lognormal function, the following equation can be used:

$$\int_0^{\rho_i} \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}(\rho/\sigma)^2} \, 2\pi \, \rho \, d\rho = \frac{N_i}{N_T} \qquad (2)$$

Equation (2) can be solved for ρ_i. Letting $z = \rho/\sigma$ and substituting yields:

$$\int_0^{\rho_i/\sigma} \frac{1}{2\rho\sigma^2} e^{-\frac{1}{2} z^2} \, 2\pi \, z \, \sigma \, \sigma dz = \frac{N_i}{N_T} \quad .$$

This reduces to

$$- \int_0^{\rho_i/\sigma} (-z) \, e^{-\frac{1}{2} z^2} \, dz = \frac{N_i}{N_T} \quad .$$

Integrating over the limits of integration yields:

$$1 - e^{-\frac{1}{2}(\rho_i/\sigma)^2} = \frac{N_i}{N_T}$$

which implies

$$-\frac{1}{2} (\rho_i/\sigma)^2 = \ln \left(1 - \frac{N_i}{N_T}\right) \quad .$$

It can be shown that $\sigma^2 = CEP^2/(2 \ln 2)$ so

$$\rho_i/CEP = \frac{-\ln(1 - N_i/N_T)}{\ln 2} \qquad (3)$$

Now $<\rho_i>$ can be found.

$$<\rho_i> = \frac{\displaystyle\int_{\rho_{i-1}}^{\rho_i} \rho \, \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}(\rho/\sigma)^2} 2\pi\rho d\rho}{\displaystyle\int_{\rho_{i-1}}^{\rho_i} \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}(\rho/\sigma)^2} 2\pi\rho d\rho}$$

The numerator equals $\dfrac{N_i}{N_T}$, so

$$<\rho_i> = N_T/N_i \int_{\rho_{i-1}}^{\rho_i} \rho \, \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}(\rho/\sigma)^2} 2\pi\rho d\rho \quad .$$

Letting $z = \rho/\sigma$ implies $dz = \dfrac{d\rho}{\sigma}$ and when $\rho = \rho_i$, $z = \rho_i/\sigma$.

$$<\rho_i> = N_T/N_i \int_{\rho_{i-1}/\sigma}^{\rho_i/\sigma} z\sigma \, \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}z^2} 2\pi \, z\sigma \, \sigma dz$$

$$<\rho_i> = N_T/N_i \, \sigma \int_{\rho_{i-1}/\sigma}^{\rho_i/\sigma} z \, e^{-\frac{1}{2}z^2} z dz \quad .$$

Letting $u = z$ and $dv = e^{-\frac{1}{2}z^2} z dz$ and integrating by parts gives

$$\langle \rho_i \rangle = \frac{N_T \sigma}{N_i} \{ [z\, e^{-\frac{1}{2}z^2}]_{\rho_{i-1}/\sigma}^{\rho_i/\sigma} - \int_{\rho_{i-1}/\sigma}^{\rho_i/\sigma} e^{-\frac{1}{2}z^2}\, dz \}$$

$$\langle \rho_i \rangle = \frac{N_T \sigma}{N_i} \{ [z\, e^{-\frac{1}{2}z^2}]_{\rho_{i-1}/\sigma}^{\rho_i/\sigma} - \sqrt{2\pi} \, [\int_{-\infty}^{\rho_i/\sigma} \frac{1}{\sqrt{2\pi}}$$

$$e^{-\frac{1}{2}z^2}\, dz \; - \; \int_{-\infty}^{\rho_{i-1}/\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, dz] \}$$

Therefore,

$$\langle \rho_i \rangle = \frac{N_T \sigma}{N_i} \{ [z\, e^{-\frac{1}{2}z^2}]_{\rho_{i-1}/\sigma}^{\rho_i/\sigma} - \sqrt{2\pi} \, [\Phi(\rho_i/\sigma)$$

$$- \; \Phi(\rho_{i-1}/\sigma)] \}$$

where Φ represents the cumulative normal distribution. Substituting for σ where $\sigma = \frac{CEP}{\sqrt{2\ln 2}}$ gives

$$\langle \rho_i \rangle = \frac{N_T}{N_i} \frac{CEP}{\sqrt{2\ln 2}} \{ \frac{\rho_i \sqrt{2\ln 2}}{CEP} e^{-\ln 2 (\rho_i/CEP)^2}$$

$$- \; \frac{\sqrt{2\ln 2}}{CEP} e^{-\ln 2 (\rho_{i-1}/CEP)}$$

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$$-\sqrt{2\pi}\ [\Phi(\frac{\sqrt{2\ell n^2}}{CEP}\ \rho_i)\ -\ \Phi(\frac{\sqrt{2\ell n^2}}{CEP}\ \rho_{i-1})]\}$$

Simplifying,

$$\frac{<\rho_i>}{CEP}\ =\ -\frac{N_T}{N_i}\ \{\frac{\rho_i}{CEP}\ e^{-\ell n^2(\frac{\rho_i}{CEP})^2}\ -\ \frac{\rho_{i-1}}{CEP}\ e^{-\ell n^2(\frac{\rho_i}{CEP})^2}$$

$$-\ \frac{\sqrt{2\pi}}{\sqrt{2\ell n^2}}[\Phi(\frac{\sqrt{2\ell n^2}}{CEP}\ \rho_i)\ -\ \Phi(\frac{\sqrt{2\ell n^2}}{CEP}\ \rho_{i-1})]\} \qquad (4)$$

To find $<\phi_i>$, the number of cells in a ring must be divided into 360°. This will provide an equal area in each cell of a particular ring. The following values for the variables of the ten cell model can be found (Table XXII).

Recalling Eq (1),

$$r_i^2\ =\ x^2\ +\ <\rho_i>^2\ -\ 2x\ <\rho_i>\ \cos\ <\phi_i> \qquad , \qquad (1)$$

it can be seen that the data presented in Table XXI is not usable in Eq (1). However, by dividing both sides of the equation by $(CEP)^2$, the equation can be written as follows:

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TABLE XXII

Ten Cell Model Values

Ring Nr. i	Cell	Cells in Ring i	Cells Inside N_i	ρ_i/CEP	$\dfrac{\langle\rho_i\rangle}{\text{CEP}}$	$\langle\phi_i\rangle$
1	1	1	1	.39	0	N/A
2	2	4	5	1.00	.711	45°
2	3	4	5	1.00	.711	135°
2	4	4	5	1.00	.711	225°
2	5	4	5	1.00	.711	315°
3	6	5	10	∞	1.51	36°
3	7	5	10	∞	1.51	108°
3	8	5	10	∞	1.51	180°
3	9	5	10	∞	1.51	252°
3	10	5	10	∞		324°

$$\left(\frac{r_i}{\text{CEP}}\right)^2 = \left(\frac{X}{\text{CEP}}\right)^2 + \left(\frac{\langle\rho_i\rangle}{\text{CEP}}\right)^2 - \frac{2 \times \langle\rho_i\rangle \cos \langle\phi_i\rangle}{\text{CEP}^2} \quad (5)$$

Simplifying,

$$r_i = \text{CEP}\left(\sqrt{\left(\frac{X}{\text{CEP}}\right)^2 + \left(\frac{\langle\rho_i\rangle}{\text{CEP}}\right)^2 - \frac{2 \times \langle\rho_i\rangle \cos \langle\phi_i\rangle}{\text{CEP}^2}}\right) \quad .$$

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r_i can be calculated using data from Table XXI (Ref 2).

It was previously shown that

$$PK(x) = \frac{1}{N_T} \sum_{i=1}^{N_T} PD(r_i) \quad .$$

To find $PK(x)$, $PD(r_i)$ must be determined. The probability of damage function can be expressed as the complementary cumulative lognormal distribution

$$PD(r) = 1 - \int_0^r \frac{1}{\sqrt{2\rho} \, \beta r'} e^{-\frac{1}{2}(\frac{\ln r' - \alpha}{\beta})} dr'$$

(Ref 5:5). Using this relationship,

$$PD(r_i) = \int_0^{r_i} \frac{1}{\sqrt{2\pi} \, \beta} r' e^{-\frac{1}{2}(\frac{\ln \theta' - \alpha}{\beta})} dr' \quad .$$

Letting $z' = \frac{\ln r' - \alpha}{\beta}$ and $dz' = \frac{1}{r'\beta} dr'$ give

$$PD(r_i) = \int_{-\infty}^{z_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z')^2} dz' \quad .$$

$$PD(r_i) = \Phi(z_i) \quad \text{where} \quad z_i = \frac{\ln r_i - \alpha}{\beta} \quad .$$

It has been shown that α and β are constants and can be found using the following equations.

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$$\alpha = \tfrac{1}{2} \ln\left[(RSS)(RSK)\right]$$

$$\beta = \frac{1}{2z_{SK}} \ln\left(RSS/RSK\right)$$

where RSS is the sure-safe range and RSK is the sure-kill range. z_{SK} = 2.054 when the sure-kill range is set at probability of kill level of 98 percent (Ref 5:7-8).

The values of PK(x) can be computed. The required inputs are the yield and CEP of the attacking weapon, the sure-safe and sure-kill ranges, and the distance of the target from DGZ. Given these inputs, the ten cell model can determine the probability of kill of a weapon against a target.

APPENDIX C

Sure-Safe and Sure-Kill Ranges
of RV When Subjected to Neutron Fluence

Sure-safe and sure-kill neutron fluence levels of the attacking RV are required to calculate the sure-safe and sure-kill ranges of the RVs. Neutron fluences of 10^{13} and 10^{17} neutrons per square centimeter (n/cm^2) were chosen as the sure-safe and sure-kill fluence levels of the RVs. The actual fluences depend on the characteristics of the RV.

To compute the desired ranges, the number of neutrons produced per kiloton yield of the interceptors must be known. The neutron source can be properly defined only by considering the actual design of the specific weapon (Ref 11: 363). Since a thermonuclear device produces the greatest number of neutrons per kiloton, the interceptor warhead will be assumed to be thermonuclear. In general, the neutrons per KT yield of a thermonuclear device is approximately 3.16×10^{23} neutrons per kiloton (Ref 2).

Figure 7 shows the $4\pi R^2$ neutron fluence as a function of the mass integral where R is the slant range from burst point to target. The mass integral (MI) measures the amount of matter a neutron must pass through to reach its target. An equation which fits this graph is:
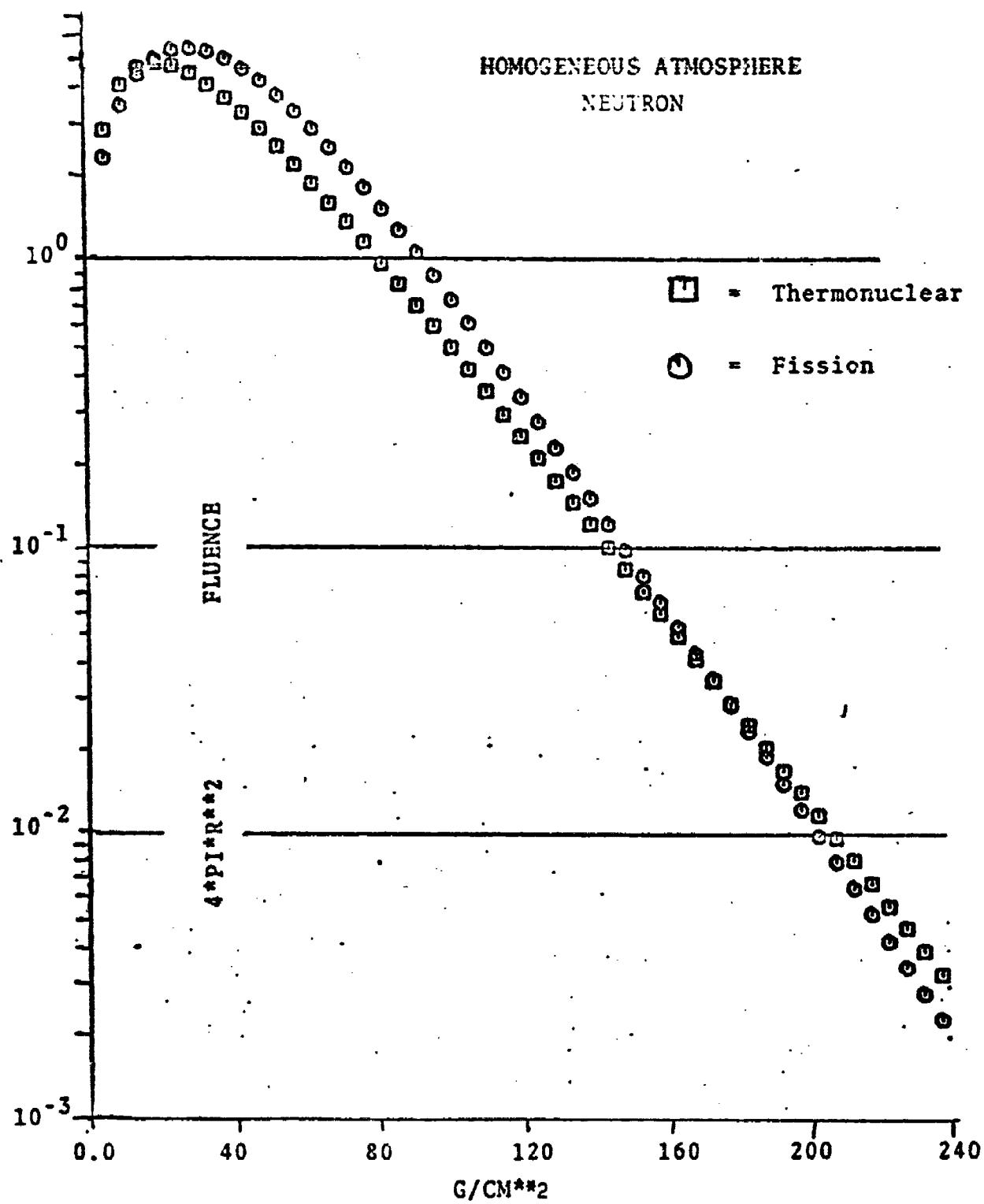
Figure 7. $4\pi R^2$ Neutron Fluence
For Fission and Thermo-
nuclear Sources (Ref 9:55)

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$$\ln(4\pi R^2 \text{ Dose}) = -6.775 + .5269 \times 10^{-2}(\text{MI})$$

$$- .54364 \times 10^{-5}(\text{MI})^2 - .21468 \times 10^{-3}(\text{MI})^{3/2}$$

$$-3.8214(\text{MI})^{1/2} + 10.875(\text{MI})^{1/3}$$

$$- 1.3975 \ (\ln(\text{MI})) \ .$$

(Ref 9:51-55)

A homogeneous atmosphere with a density of 0.001 gram per cubic centimeter was assumed. This assumption is reasonable for the altitude of the intercept. This quantity is used to calculate the MI and

$$\text{MI} = R \ \rho$$

where R is the distance between the target and the burst point, and ρ is the density of the atmosphere.

With the above information, the following interactive program is an iterative process used to calculate the sure-safe and sure-kill ranges.

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```
PRINT"*************************************************************
PRINT"THE FOLLOWING PROGRAM IS AN ....    E PROGRAM.  ...
PRINT"MUST ENTER THE DISTANCE IN ....     BURST POINT TO ....
PRINT"FOR THE GIVEN ATTACK.  tHE PROGRAM COMPUTES THE NEUTRON
PRINT"FLUENCE THE TARGET EXPERIENCES.  WHEN THE FLUENCE IS
PRINT"EQUAL TO THE DESIRED LEVEL, THE .... NOW HAS THE DESIRED
PRINT"SURE-KILL OR SURE-SAFE RANGE."
PRINT"ENTER THE SLANT IN FEET FROM BURST POINT TO TARGET"
INPUT SR
 SR=SR*30.48
 PRINT"ENTER THE YIELD IN KILOTONS OF THE INTERCEPTOR"
 INPUT KT
 MI=.001*SR
 J=-6.775+.5269E-2*MI-.54364E-5*MI*MI-.21468E-3*(MI[1.5)
 K=J-3.8214*(MI[.5)+10.875*(MI[(1/3))-1.3975*LOG(MI)
 DOSE=EXP(K)*3.16E+23*KT/(4.*3.1416*(SR[2))
 PRINT"THE SLANT RANGE=",SR/30.48
 PRINT"THE YIELD OF THE INTERCEPTOR=",KT
 PRINT"THE DOSE ON RV AT GIVEN SLANT RANGE "
 PRINT"FOR THE GIVEN YIELD=",DOSE
 END
 PRINT"*************************************************************
```

APPENDIX D

A Listing of the Computer Model

and

The Q-GERT Network

Q-GERT MODEL


```
*** INPUT CARDS ***

GEN,MOORE,MODEL2,11,21,19??,5,?,1,,???,,,,8*
SOU,1,,,1,A,M*
*
*NODE 1 CREATES ATTACKING RVS AND
*TARGETS EACH ON A SHELTER
*
VAS,1,1,UF,1,2,IN,1,3,IN,2,,+,UF,2,?,UF,3*
ACT,1,1,,,1/AFR1/A_,(9)A2._T.\1*
ACT,1,2,(9)A2.LE.A5*
ACT,1,9 ,DO,1,(_)A2.GT.A5
SOU,7,,,1,A,M*
*
*NODE 7 RANDOMLY ASSIGNS THE JJ AND THE
*MX TO A SHELTER
*
VAS,7,2,JF,_,3,UF,3,4,JF,2,5,JF,3,6,UF,5*
ACT,7,8,(9)A2.LE.A5*
ACT,7,9,(9)A3.LE.A3*
ACT,7,1 ,(9)A2.GT.A5*
ACT,7,11,(9)A3.GT.A3*
REG,2,1,1*
ACT,2,5*
ACT,2,6*
QUE,5/MXQJE,(10)12*
QUE,6/DUQJE,(10)13*
QUE,8/MXSHEL,(1 )12*
QUE,9/DUSHEL,(1 )13*
REG,10,1,1*
VAS,13,2,DO,_*
REG,11,1,1,*
VAS,11,3,DO, *
MAT,12,2,_/1_,8*
MAT,13,3,_/1_,9*
REG,14,1,1*
VAS,14,2,DO,1*
REG,15,1,1*
VAS,15,3,DO,1*
ACT,1_,1!
ACT,11,1_
ACT,14,15 .
ACT,15,17 .
REG,15,1,1*
*
```

```
*
*NODE 16 CALCULATES TOTAL NUMBER
*OF RVS AIMED AT THE MX SHELTER
*
VAS,16,7,JF,7*
REG,17,1,1*
*
*NODE 17 CALCULATES TOTAL NUMBER
*OF RVS AIMED AT THE DU SHELTER
*
VAS,17,8,JF,8*
ACT,16,18,CO,1*
ACT,17,2 ,CO,1*
REG,18,1,1,A*
ACT,18,19,(S)A2..T.A7*
ACT,18,6 ,(S)A7.EQ. *
ACT,18,21,UN,1,(3)A7.GT.0*
VAS,19,2,IN,1,4,DO,1*
REG,20,1,1,A*
VAS,20,3,IN,1,4,DO,2*
ACT,20,20,(S)A3..T.A8*
ACT,20,A1,(S)A8.EQ. *
ACT,20,21,UN,1,(3)A8.GT.0
SIN,31/MXALIVE,1,1,D,1*
STA,31/DUALIVE,1,1,D,1*
STA,31/RVCOUNT,1,1,D,1*
REG,21,1,1*
*
*NODE 21 ASSIGNS EACH RV AIMED AT THE
*DU OR MX SHELTER A RANDOM NUMBER
*
VAS,21,5,JF,9*
ACT,21,22
QUE,22/RVARRIVE,0,,0,B/5*
*
*Q-NODE 22 DETERMINES SEQUENCE
*OF ATTACK FOR THE RVS
*
ACT,22,23,CO,30*
REG,23,1,1,A*
ACT,23,24,(S)A4.EQ.1*
ACT,23,25,(S)A4.EQ.2*
REG,24,1,1,B*
*
*NODE 24 LAUNCHES AN INTERCEPTOR,
*IF AVAILABLE, AT AN RV ATTACKING
*THE MX SHELTER
*
VAS,24,2,JF,10,0,JF,11*
ACT,24,26,(F)2*
ACT,24,30,(F)3*
REG,25,1,1,B*
```

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```
*
*NODE 25 LAUNCHES AN INTERCEPTOR,
*IF AVAILABLE, AT AN RV ATTACKING
*THE DU SHELTER
*
VAS,25,2,JF,1.,3,JF,11'
ACT,25,92,(E)2*
ACT,25,26,(E)3*
REG,25,1,1,F*
VAS,26,2,JF,12,3,JF,13'
ACT,26,27,(E)2*
ACT,26,28,(E)3*
STA,27/DUDEAD,1,1,D,I-
*
*STATISTICS NODE 27 RELEASED
*IF DU IS DESTROYED
STA,28/DUALIVE,1,1,D,I-
*
*STATISTICS NODE 28 RELEASED
*IF DU IS NOT DESTROYED
*
STA,32/DUALIVE,1,1,D,I'
REG,28,1,1*
ACT,28,31
REG,30,1,1*
ACT,30,31
REG,31,1,1,4*
*
*NODE 31 DETERMINES NUMBER OF
*RVS REACHING MX SHELTER
*
VAS,31,4,JF,12,5,JF,14*
ACT,31,33,(9)A5.EQ.1*
ACT,31,32,(S)A5.EQ.2*
STA,32/RVCOUNT,1,1,D,I'
REG,33,1,1,F*
VAS,33,5,JF,15,6,JF,16*
ACT,33,7.,(6)5*
ACT,33,71,(6)6*
SIN,70/MXDEAD,1,1,D,I*
*
*SINK NODE 70 RELEASED IF THE
*MX IS DESTROYED
*
SIN,71/MXALIVE,1,1,D,I'
*
*SINK NODE 71 RELEASED IF THE
*MX SURVIVES THE ATTACK
*
PAR,1,,1,0,*
FIN*
```

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```
          FUNCTION UF(IFN)
          COMMON/QVAR/NDE,NETBU(1J0),NREL(100),NRELP(100),NRELZ(100),
         &NRUN,NRUNS,ATC(10),PARAM(10,4),IDEG,TNOW
          COMMON/RANG/SSR(10U),SKR(100),STR
          COMMON/VAR1/AVKT,SSPSI,SKPSI,DEFIN,INKT,RSS,RSK
          COMMON/VAR2/CEPRV,CEPIN,PD,ATTRV,CEP,X,OJIN
          GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16), IFN
C***      THE FOLLOWING FUNCTIONS RETURN VALUES TO ATTRIBUTES
C             OF THE Q-GERT MODEL
C***      ASSIGN NUMBER OF ATTACKING RVS(ATT1)
1         UF=ATTRV
          RETURN
C
C***      DETERMINE MINIMUM NUMBER OF RVS ATTACKING EACH
C             SHELTER(ATT6)
C
2         UF=INT(ATTRV/23.)
          RETURN
C
C***      DETERMINE NUMBER OF RVS RANDOMLY AIMED AT
C             SHELTERS(ATT5)
C
3         UF=ATTRV-INT(ATTRV/23.)*23.
          RETURN
C
C***      RANDOMLY ASSIGN MX TO A SHELTER(ATT2)
C
4         RN=DRAND(1)
          UF=INT(RN*23)+1.
          RETURN
C
C***      RANDOMLY ASSIGN DU TO A SHELTER(ATT3)
C
5         RN=DRAND(2)
          UF=INT(RN*23)+24
          RETURN
C
C***      DETERMINE NUMBER OF RVS RANDOMLY AIMED
C             AT SHELTERS(ATT6)
C
6         UF=ATTRV-INT(ATTRV/23.)*23.+23.
          RETURN
C
C***      DETERMINE NUMBER OF RVS ATTACKING MX(ATT7)
C
7         ATT2=GATRB(2)
          ATT4=GATRB(4)
          UF=ATT2+ATT4
          RETURN
```

```
C
C***    DETERMINE NUMBER OF RVS ATTACKING DU(ATT3)
C
8       ATT3=SATRB(3)
        ATT4=SATRB(4)
        UF=ATT3+ETT4
        RETURN
C
C***  RANDOMLY ASSIGN A NUMBER BETWEEN ONE AND ONE
C     THOUSAND TO EACH RV(ATT5)
C
9       RN=DRAND(3)
        JF=INT(RN*1000.)+1.
        RETURN
C
C***    DETERMINE PROBABILITY OF DESTROYING ATTACKING RV
C        WITH AN INTERCEPTOR(ATT2)
C
C       CHECK TO DETERMINE IF DU HAS BEEN DESTROYED
10      IF(NFO(27).GT...)GO TO 100
C       DETERMINE IF REMAINING INTERCEPTORS WILL DEFEND DU
        ATT4=SATRB(4)
        IF(ATT4.EQ.2 .AND. DEFIN.LE.STR)GO TO 10.
C       DETERMINE AVAILIABILITY OF INTERCEPTORS
        IF(DEFIN.EQ..)GO TO 100
C       DETERMINE SURE SAFE AND SURE KILL RANGES OF INTERCEPTOR
C       RSS IS SURE SAFE RANGE AND RSK IS SURE KILL RANGE
C       SSR AND SSK ARE THE ARRAYS FOR THESE RANGES AND
C          THE RANGES DEPEND ON INTERCEPTOR YIELD
        RSS=SSR(INKT)
        RSK=SKR(INKT)
C       FIRE INTERCEPTOR
        DEFIN=DEFIN-1.
        DEP=DEPIN
        X=1.
C       USES TEN CELL MODEL TO CALCULATE PK
        CALL JS(1)
        JF=PP
        RETURN
100     JF=1.
        RETURN
C
C***    DETERMINE PROBABILITY OF INTERCEPTOR MISSING RV(ATT3)
C
11      ATT2=SATRB(2)
        JF=1.-ATT2
        RETURN
```

```
C
C***     CALCULATE PROBABILITY OF ONE RV DESTROYING A SHELTER
C        CALCULATE THE PK DUE TO OVERPRESSURE
C           RSS AND RSK ARE SURE SAFE AND SURE KILL RANGES
C
12       RSK=EXP(7.967/(SKPSI**.072))*(RVKT*2)**(1./3.)
         RSS=EXP(7.967/(SSPSI**.072))*(RVKT*2)**(1./3.)
         K=0,
         DEP=DEPRV
         CALL JS(1)
         PKOP=PJ
C        CALCULATE PK FROM CRATERING
         CRAD=5L.*(RVKT*.3)
         SIG=DEP/SQRT(2*ALOG(2.))
         ZRA=CRAD/SIG
         PKCR=2*(1.-(1./(2.*(1+.19685*ZRA+.11519+*ZRA*2+
     S.     ZRA*ZRA**3+.19527*ZRA**4)**4))-.5)
C        CALCULATE PROBABILITY OF KILL FROM ONE RV
         PK=PKOP+(1-FKOP)*PKCR
         JF=PK
         RETURN
C
C***     CALCULATE PROBABILITY OF SHELTER SURVIVING ONE RV
C
13       ATT2=GATRB(2)
         JF=1.-ATT2
         RETURN
C
C***     ACCOUNT FOR RVS AIMED AT MX(ATT5)
C
14       AA=NTC(29)+NTC(30)
         ATT7=GATRB(7)
         IF(AA.EQ.ATT7)JF=1
         IF(AA.NE.ATT7)UF=2
         RETURN
C
C***     CALCULATE PROBABILITY OF MX DESTRUCTION(ATT4)
C
15       ATT4=GATRB(4)
         JF=1.-(1.-ATT4)**NTC(30)
         RETURN
C
C***     CALCULATE PROBABILITY OF MX SURVIVAL(ATT5)
C
16       ATT5=GATRB(5)
         JF=1-ATT5
         RETURN
         END
```

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```
      SUBROUTINE UI
      COMMON/OVAR/NDF,NFTRU(1..),NREL(1..),NRELP(1..),NREL2(1..),
     $NRUN,NRUNS,NTC(1..),PARAM(1..,.),ISEG,TNOW
      COMMON/RANG/SSR(1..),SKR(1..),STR
      COMMON/VAR1/RVKT,SSPSI,SKPSI,DEFIN,INKT,PSE,RSK
      COMMON/VAR2/CEPRV,CEPIN,PD,ATTRV,CEP,X,DJTH
C***  THIS SUBROUTINE ALLOWS THE USER TO INPUT PARAMETERS
C***  ENTER RV YIELD
      RVKT=125
C***  ENTER RV CEP
      CEPRV=1000
C***  ENTER NUMBER OF ATTACKING RVS
      ATTRV=69
C***  ENTER SHELTER SURE SAFE PSI
      SSPSI=25.
C***  ENTER SHELTER SURE KILL PSI
      SKPSI=75.
C***  ENTER THE STRATEGY FOR THE INTERCEPTORS
C     THE STRATEGY IS A NUMBER(1,2 OR 3) AND REPRESENTS
C     THE NUMBER OF INTERCEPTORS THAT WILL BE USED FOR
C     RK SHELTER DEFENSE ONLY.
      STR=1
C***  ENTER INTERCEPTOR YIELD
      INKT=5
C***  ENTER NUMBER OF DEFENSIVE INTERCEPTORS
      DEFIN=3
      DJIN=3
C***  ENTER INTERCEPTOR CEP
      CEPIN=300
C***  ENTER THE ARRAYS FOR SURE SAFE AND SURE KILL
C        RANGES OF THE RVS
      DATA SSR/1..*0../
      DATA SKR/1..*0../
      DATA(SKR(I),I=1,50)/17.6,25.3,32.5,39.1,45.8,
     $51.3,58.5,65.5,69.3,73.0,74.5,79.1,83.0,87.6,
     $91.3,95.0,10..,104.,108.,111.,115.,119.,123.,
     $125.,131.,134.,137.,141.,144.,148.,151.,154.,
     $158.,161.,164.,168.,171.,174.,177.,180.,183.,
     $185.,188.,192.,195.,198.,2.1.,2.4.,2.7.,21..../
      DATA(SKR(I),I=55,100,5)/224.,277.,250.,263.,270.,
     $287.,299.,301.,321.,331./
      DATA(SSR(I),I=1,50)/213..,2433.,2723.,2685..,3017.,3113.,
     $3213.,3256.,330 5.,3618.,347..,3525..3574..4519.,
     $3353.,77...,3737.,3771.,38..,3535.,3565.,7559..,
     $3792..,3506.,3973.,3997.,.023.,..15.,.05.,.186..,
     $3.1,5.,4128.,4145.,4134..4132.,4133.,.215.,.237.,
     $5.243.,.254.,.261.,4230.,.311.,.325.,.339.,.313.,
     $3.357.,43...,4393.,44.6./
      DATA(SSR(I),I=55,100,5)/.43.,.522.,.573.,.62..,.551..,
     $.7.5.,.7...,.781.,.816.,.649./
      RETURN
      END
```

```
      SUBROUTINE US(IFN)
      DIMENSION PI(1 ),THETA(10),R1(1   2(1 ),ZA(1 ),PDZ(1 ),PDR(1 )
      COMMON/QVAR/NO ,NFTBU(1 ),NREL(1  ),NRELP(1  ),NREL2(1  ),
     $NRJN,NRUNS,NTC(1  ),PARAM(1 ,4),TBES,TNOW
      COMMON/RANG/SSR(1 ),SKR(1  ),STR
      COMMON/VAR1/RVKT,SSPSI,SKPSI,DEFAN,INKT,RSS,PSK
      COMMON/VAR2/DEPRV,CEPIN,PD,ATTRV,CEP,X,DUIN
C**   CALCULATES FK USING TEN CELL MODEL
C        FOR EXPLANATION OF TEN CELL
C        MODEL SEE APPENDIX B
      GO TO (1), IFN
1     PSK=2. 1.
      ALPHA=.5*ALOG(RSS*RSK)
      BETA=1./(2.*PSK) (ALOG(RSS/RSK))
      DATA(PI(I),I=1,1 )/. ,4*.711,5*1.31/
      DATA(THETA(J),J=1,10)/. ,45.,135.,235.,315.,36.,1 6.,
     $18 .,252.,32 ./
      DO 3  I=1,1
      PI(I)=SQRT(P1(I) *2+(X/CEP)  2-2*X*PI(I)/CEP*
     $COS)(THETA(I)))*CEP
31    CONTINUE
      DO 4  K=1,1
      IF(R1(K).EQ. .AND. X.EQ. )GO TO 35
      Z(K)=(ALOG(F1(K))-ALPHA)/BETA
      GO TO 4
35    Z(K)=-1  .
4     CONTINUE
      PDX= .
      DO 7  L=1,1
      ZA(L)=ABS(Z(L))
      PDZ(L)=1./(2*(1.+.196854*ZA(L)+.115194*ZA(L)**2+
     $ .00344 *ZA(L)* 3+.019527*ZA(L)**.1/*4)
      IF(Z(L).GT. .)GO TO 50
      PDR(L)=1.-PDZ(L)
      GO TO 60
5     PDR(L)=PDZ(L)
50    PDX=PDX+PDR(L)
7     CONTINUE
      PD=PDX/1.
      RETURN
      END
```

USER OUTPUT

```
      SUBROUTINE UO
      COMMON/OVAR/NDE,NFTBU(1..),NREL(1..),NRELP(1..),NFEL2(1..),
     &NRUN,NRUNS,NTC(1..),PARAM(1..,4),IBEG,TNOW
      COMMON/RANG/SSF(1..),SKR(1..),STR
      COMMON/VAR1/AVKT,SSPSI,SKPSI,DEFIN,INKT,RSS,RSK
      COMMON/VAR2/CEPRV,CEPIN,PD,ATTRV,CEP,X,DJIN
      COMMON/DELTA/ALIVE,DEAD
C***  THIS SUBROUTINE PROVIDES A PRINTOUT OF MODEL
C        PARAMETERS AND PERCENT OF MX DESTROYED
C        AND MX SURVIVING
      DATA DEAD,ALIVE/...,".../
      DEAD=DEAD+NTC(..)
      ALIVE=ALIVE+NTC(71)+NTC(66)
      IF(NRUN.EQ.NRUNS)GO TO 2..
      RETURN
2..   ALIVE=ALIVE/6.
      DEAD=DEAD/6.
      PRINT*,"THE NUMBER OF ATTACKING RVS IS  ",ATTRV
      PRINT*,"THE YIELD OF THE RVS IS          ",FVKT
      PRINT*,"THE CEP OF THE RVS IS            ",CEFRV
      PRINT*,"THE INTERCEPTOR STRATEGY IS      ",STR
      PRINT*,"THE NUMBER OF INTERCEPTORS IS    ",DUIN
      PRINT*,"THE YIELD OF THE INTERCEPTOR IS ",INKT
      PRINT*,"THE CEP OF THE INTERCEPTOR IS    ",CEPIN
      PRINT*,"THE SURE-SAFE PSI LEVEL IS       ",SSPSI
      PRINT*,"THE SURE-KILL PSI LEVEL IS       ",SKPSI
      PRINT*,"THE PERCENT KILL OF MX=          ",DEAD
      PRINT*,"THE PERCENT LIVE OF MX=          ",ALIVE
      RETURN
      END
```
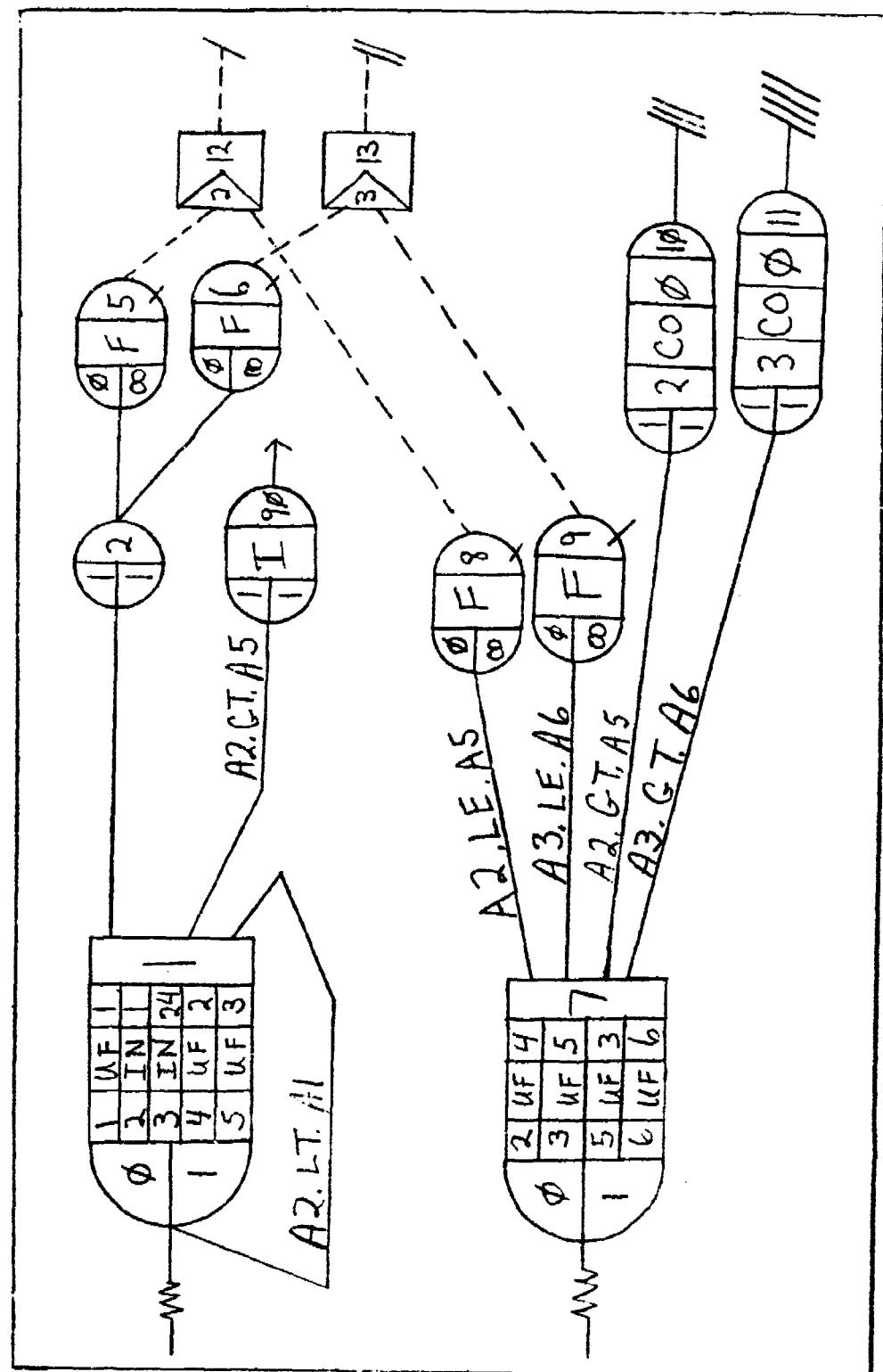
Figure 8-1. Q-GERT Network
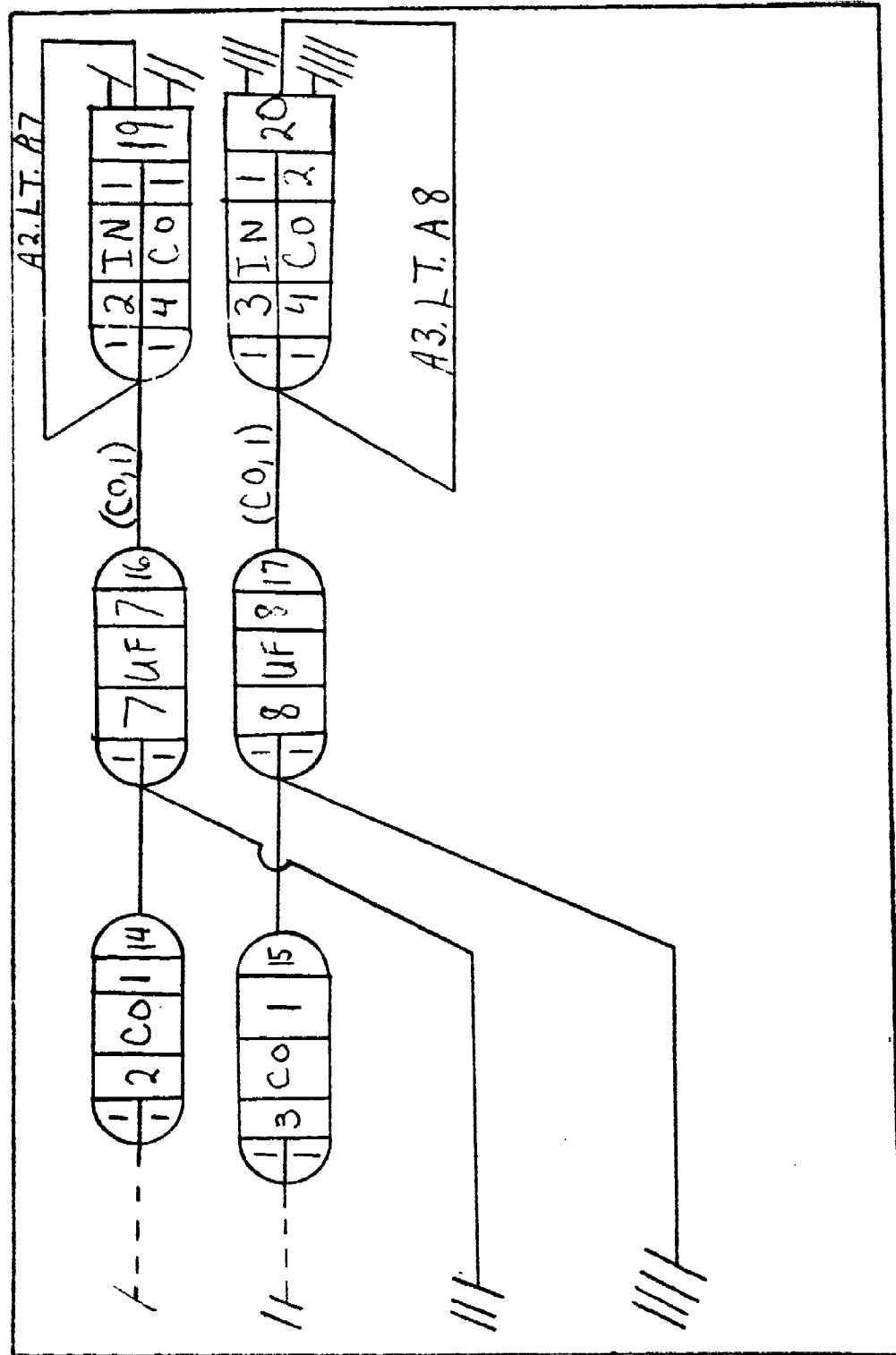
Figure 8-2. Q-GERT Network

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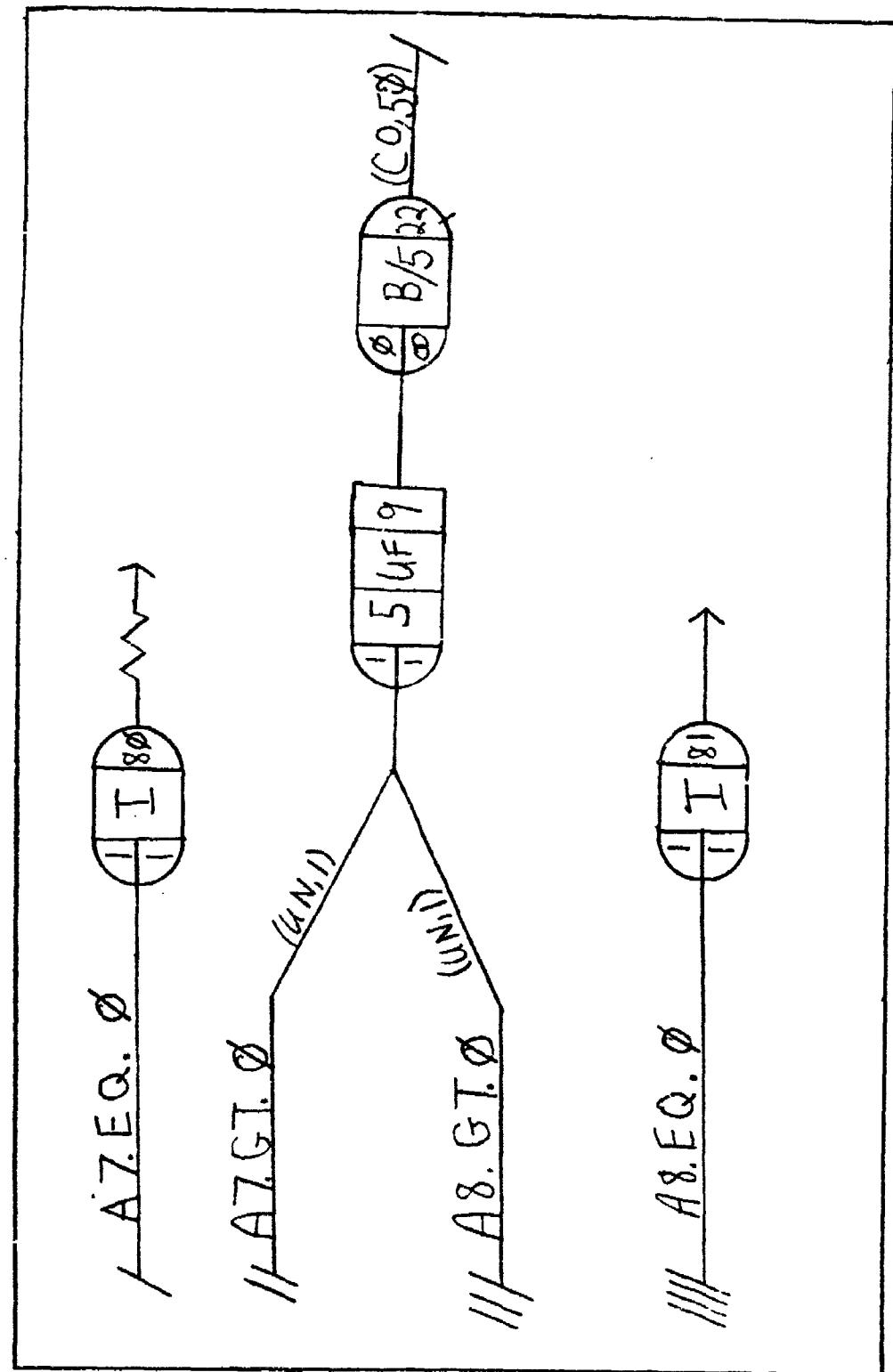
Figure 8-3. Q-GERT Network
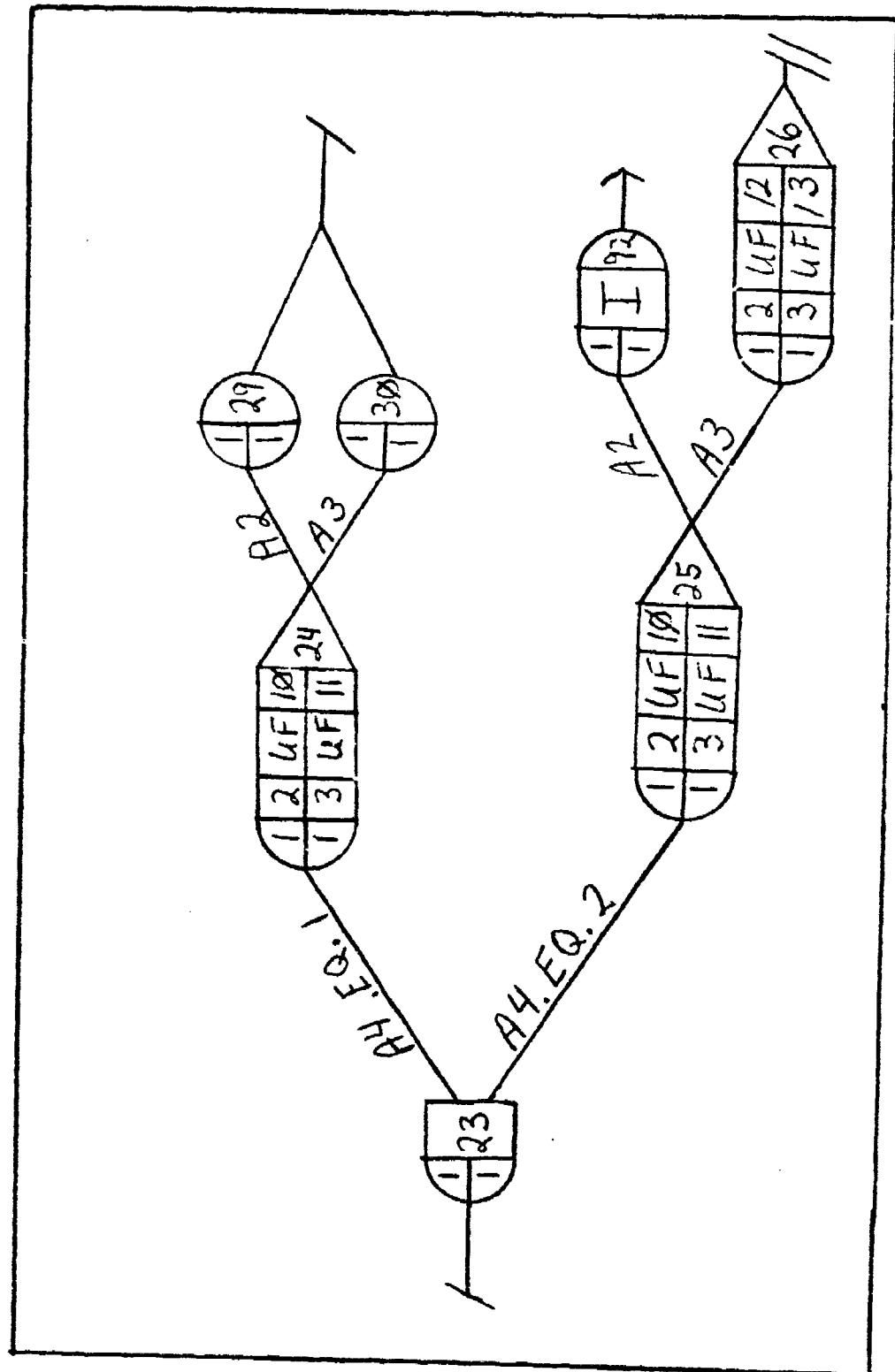
Figure 8-4. Q-GERT Network
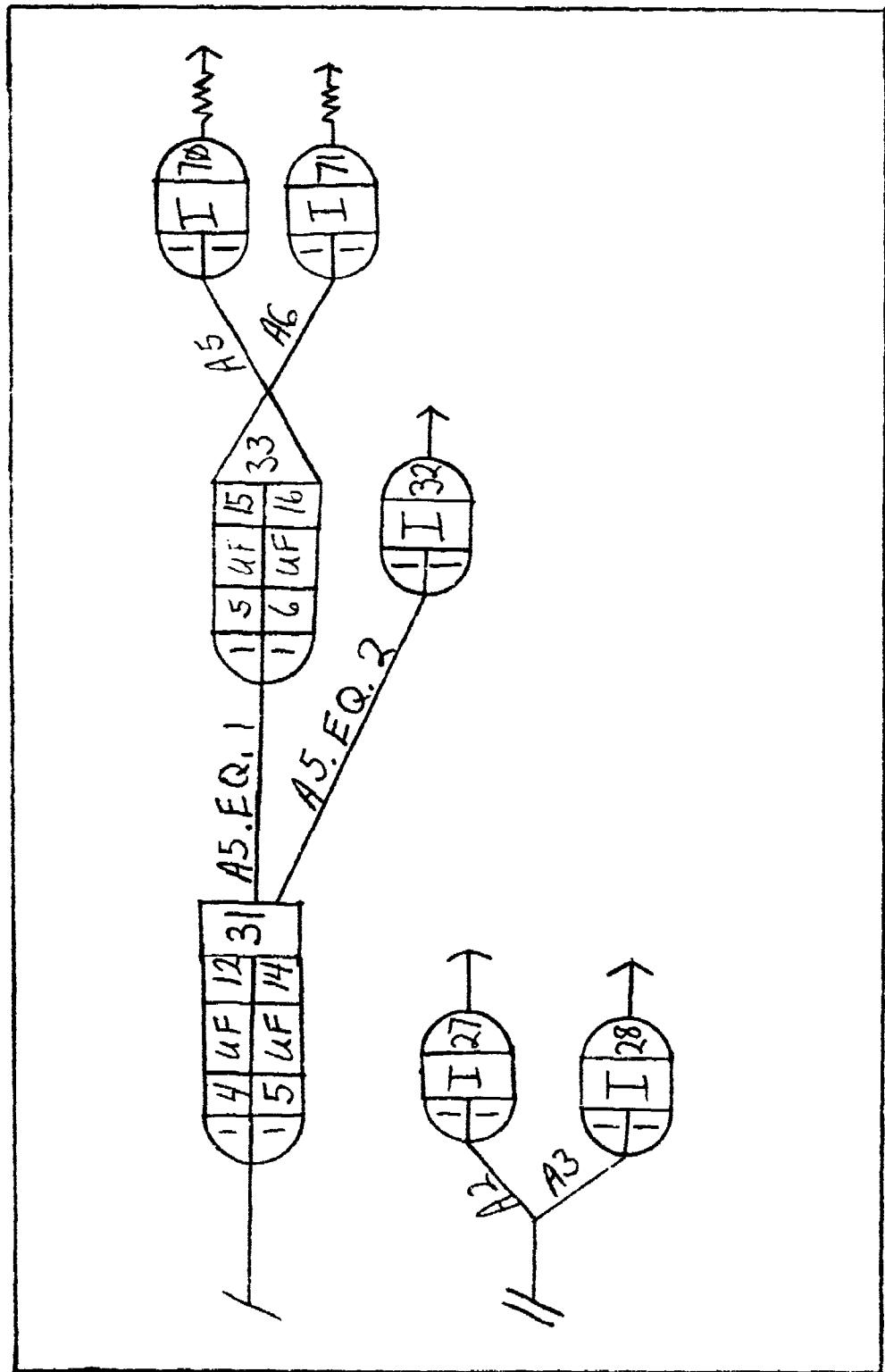
Figure 8-5. Q-GERT Network

Vita

James Thomas Moore was born on 7 April 1952 at Offutt
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commission in the USAF through Officer's Training School
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Deputy Missile Combat Crew Commander, missile instructor,
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<div align="right">

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